Nâme	Sâlmân Khân
i-D	5778
Subject	Calculus & Analyticâl Geometry
Date	19-08-2020

Q=1 (a) Identify $\lim_{x \to 1} \frac{x^2+x-2}{x^2-x}$

Sol:

$$\lim_{x \to 1} \frac{x^2+2x-x-2}{x(x-1)}$$

$$= \lim_{x \to 1} \frac{x(x+2)-1(x+2)}{x(x-1)}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+2)}{x\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{x+2}{x} = \frac{1+2}{1} = \boxed{3}$$

Q=1 (b) Find the first order derivatives of the function $y = (3-x^2)(x^3-x+1)$

Sol:- $\frac{dy}{dx} = \frac{d}{dx}\left[(3-x^2)(x^3-x+1)\right]$

$= (3-x^2)\frac{d}{dx}(x^3-x+1) + (x^3-x+1) + \frac{d}{dx}(3-x^2)$

$= (3-x^2)(3x^2-1) + x^3-x+1\,(0-2x)$

$= 9x^2-3-3x^4+x^2-2x^4+2x^2-2x$

$= -5x^4+12x^2-2x-3$

Question given in Paper

Q-2 (a) At a given time t the body position along s-axis is given by $s = t^3 - t^2 + 9t$

(1) Find the body acceleration each time when the velocity is zero.

2) Find the body speed each time when the acceleration is zero.

Sol:- Given $s = t^3 - t^2 + 9t$

(a) $v = \frac{ds}{dt} = 3t^2 - 2t + 9$

$a = \frac{dv}{dt} = 6t - 2$

when $v = 0$ then $3t^2 - 2t + 9 = 0$

Here we will use Quadratic equation as it cannot be simplified

So $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here $a = 3$ $b = -2$ $c = 9$

$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(9)}}{2(3)}$

$\frac{2 \pm \sqrt{4 - 108}}{6} = \frac{2 \pm \sqrt{-104}}{6}$

$= 2 \pm 4.16$

So $t_1 = 2 + 4.16 = 6.16$ $t_2 = 2 - 4.16 = 2.16$

So $t_1 = 6.16$ Sec $\quad t_2 = 2.16$ sec

As $a = 6t - 2$

$a(6.16) = 6(6.16) - 2 = 34.96 \ m/sec^2$

$a(2.16) = 6(2.16) - 2 = 10.96 \ m/sec^2$

(b) when $\hat{a} = 0$ Then $6t - 2 = 0$

$t = 1/3$

As $V = 3t^2 - 2t + 9$

$V = 3(1/3)^2 - 2(1/3) + 9$

$V = 0.11 - 0.66 + 9$

$$\boxed{V = 8.45 \ m/sec}$$

Question from lecture.

③

Q=2 ⓐ At â Time t the body Position along S-axis is given by. $S = t^3 - 6t^2 + 9t$

① Find the body âcceleration eâch time when the Velocity is Zero.

(2) Find The body Speed each time when The âcceleration is Zero.

Sol:-

$$S = t^3 - 6t^2 + 9t$$

$$V = \frac{ds}{dt} = 3t^2 - 12t + 9$$

$$a = \frac{dv}{dt} = 6t - 12$$

when $V = 0$ Then $3t^2 - 12t + 9 = 0$

$$t^2 - 4t + 3 = 0$$

$$t^2 - 3t - t + 3 = 0$$

$$t(t-3) - 1(t-3) = 0$$

$$(t-3)(t-1) = 0$$

$t_1 = 3$ Sec $\quad t_2 = 1$ Sec

As $a = 6t - 12$

$a(1) = 6 - 12 = -6\,m/sec^2$

$a(3) = 6(3) - 12 = 18 - 12 = 6\,m/sec^2$

(b) when $\hat{a} = 0$ Then $6t - 12 = 0$

$t = 2$ sec

As $V = 3t^2 - 12t + 9$

$V(2) = 3(2)^2 - 12(2) + 9$

$= 12 - 24 + 9 = -3$

$|v(2)| = |-3| = 3\,m/sec$

Q:3 (a) Find the equation of tangent & normal to the curve at the given Point.

Where $x^2 - xy + y^2 = 7 \quad (2,3)$

Sol:- First find Use implicit differentiation To find $\frac{dy}{dx}$. Given That $x^2 - xy + y^2 = 7$

5

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 7$$

$$2x - \left(x\frac{dy}{dx} + y(1)\right) + 2y\cdot\frac{dy}{dx} = 0$$

$$2x - y - x\left(\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$(2y - x)\frac{dy}{dx} = -2x + y$$

$$\frac{dy}{dx} = \frac{-2x + y}{2y - x}$$

Given Point is $(x,y) = (2,3)$

$$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{-2(2)+3}{2(3)-(2)} = \frac{-1}{4}$$

The equation of tangent is

$$(y-y_1) = \frac{dy}{dx}(x-x_1)$$

$$(y-3) = \frac{-1}{4}(x-(2))$$

$$(y-3) = -1/4\,(x-2)$$

$$4(y-3) = -x+2$$

$$4y-12 = -x+2$$

$$4y-12+x-2=0$$

$$4y+x-14=0 \qquad x+4y-14=0$$

$$y-y_1 = -\frac{1}{dy/dx}(x-x_1)$$

$$(y-3) = -\left(\frac{1}{-1/4}\right)(x-2)$$

$$(y-3) = -\left(1\times\frac{4}{1}\right)(x-2)$$

$$(y-3) = 4(x-2)$$

$$(y-3) = 4x-8$$

$$y-3-4x+8=0$$

$$\boxed{-4x+y+5=0}$$