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SEC :- A

Subject :- Fluid Mechanics II

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Ans 1 (A)

Ans:-

Forced on Immersed Bodies:-

A body which is wholy immersed in a homogeneous Fluid may be subjected in two kind of Forces arising From relative motion b/w body and Fluid. These Forces are termed as drag and lift. depending on Forces either parallel or right angle to motion.

Drag Force on submerged body can have 2 components.

1) Pressure Drag(FP):-

It is equal to the integration of ~~[illegible]~~ component also direction of notion of all pressure Forces exerted on surface of body.

$$F_P = C_P \cdot \rho \cdot \frac{v^2}{2} \cdot A$$ where C_P depend on shape

2) Friction Drag:-

It is equal to integration of component of all shear stress along the surface in direction of motion.

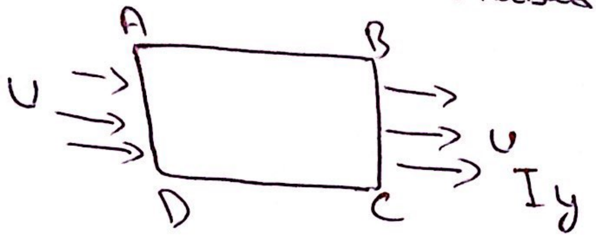
$$F_P = C_f \cdot \rho \frac{V^2}{2}(BL)$$

Friction Drag of Boundary layer:-

$$\bar{\tau}_o = \mu \frac{u\,d(n)}{\delta\, d(n)}$$

$$\bar{\tau}_o = \frac{\mu u \cdot B}{\delta}$$

Figure shows growth of boundary layer along one side of smooth plate in steady flow of incompressible fluids. Consider value where δ is the thickness of boundary layer & U is undisturbed velocity.



As we have $\Sigma F_x = 0$
where

$$F_x = \frac{\Delta P}{\Delta t} = \frac{\Delta mv}{\Delta t}$$

$$F_x = \frac{\Delta \rho . vol . v}{\Delta t} = \Delta \rho Q v$$

$$F_x = \Delta \rho Q v$$

F_x = rate of change BC + AB - AD

$$AD = \rho U (U \delta B)$$

$$BC = \rho B (u^2 dy)$$

$$AB = \rho U (UB\delta) - B \int_0^\delta u dy$$

$$F_x = \rho B \int_0^\delta u (U - u) dy \quad -(1)$$

Integrate b.s

$$F_x = \rho B U^2 \delta a$$

wher a is a function of boundary layer

Now to find shear stress

$$\tau = \frac{F_x}{A} = \frac{dF_x}{B dx} = \frac{dF_x}{B dx}$$

$$\tau_0 = \rho B U^2 \alpha \frac{d\delta}{B\,dx} = \rho U^2 \alpha \frac{d\delta}{dx}$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx} \quad -\bullet$$

Laminar Boundary layer:-

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad -①$$

$$\frac{y}{\delta} = n \Rightarrow y = \delta n$$

$$dy = \delta dn \quad -②$$

$$\frac{u}{U} = f(n)$$

$$du = U df(n) \quad -③$$

For laminar flow

$$\tau_0 = \mu \frac{du}{dy} \quad -④$$

$$\tau_0 = \mu \frac{U df(n)}{\delta d(n)}$$

$$\tau_0 = \frac{\mu U \beta}{\delta}$$

As we have

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Compare both ③

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta\, d\delta = \frac{\mu B\, dx}{\rho U \alpha}$$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + c$$

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

$$B = 1.68 \,, \alpha = 1.35$$

$$\delta = \frac{4.91 x}{\sqrt{R_x}}$$

where (R_x) is local Reynold number

As we have

$$\tau_0 = \frac{\mu U B}{\delta}$$

Put ⑥ in eq ⑤

$$\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

Now

$$F_f = B \int \tau_o \, dx$$

where $\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$

$$R_x = \frac{x U \rho}{\mu}$$

then putting value we have

$$F_f = 0.664 \sqrt{\rho \mu L U^3}$$

As we have

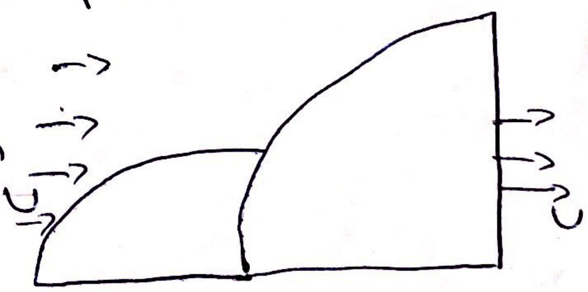
$$F_f = C_F \rho \frac{V^2}{2} B L$$

Equating b/s

$$C_F = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \boxed{\frac{1.328}{\sqrt{R_x}}}$$

Turbulent Boundary layer:-

This Fig shows the velocity distribution of boundary layer which is steeper near walls and flatter through one remainds of layer



The shear stress is greater in turbulent than in laminar

thus $\tau_o = \rho f \frac{V^2}{8}$

where V is the average velocity

to obtain relation b/w

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}} \qquad \because f = 0.028$$

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{0.028}}$$

$$U = 1.235\,V$$

$$V = \frac{U}{1.235}$$

$$f = \frac{0.316}{(R_e)^{1/4}} \qquad R_e = \left(\frac{DV}{\nu}\right)$$

$$\tau_o = f\rho\frac{V^2}{8}$$

$$\tau_o = \frac{0.316}{\left(\left(\frac{D}{\nu}\right)\left(\frac{U}{1.235}\right)\right)^{1/4}}$$

$$\tau_o = \frac{0.023\rho U^2}{\left(\frac{2\delta}{\nu}\right)^{1/4}} \quad \text{—(1)}$$

As we have general equation

$$\tau_o = \rho U^2 \alpha \frac{d\delta}{dx} \quad \text{—(2)}$$

equate (1) and (2)

$$x = 0 \quad , \quad \delta = 0$$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5}\left(\frac{\nu}{Ux}\right)^{1/5}.x$$

$$\alpha = 0.0972$$

$$\delta = \frac{0.377}{(R_x)^{2/5}}.x \quad \text{—(3)}$$

$$\tau_o = 0.0587\rho\frac{U^2}{2}\left(\frac{\nu}{Ux}\right)^{1/5}$$

Now

$$F_f = B \int_0^L \tau_0 \, dx$$

$$F_f = 0.0735 \, \rho \, \frac{U^2}{2} \left(\frac{\nu}{UL}\right)^{1/5} . BL$$

$$F_f = C_f . \rho . \frac{V^2}{2} . BL$$

$$C_f = \frac{0.0735}{(R)}$$

For $R > 10$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.52}}}$$

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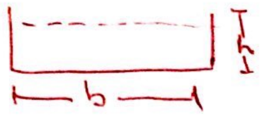
Q.no 1 (B)

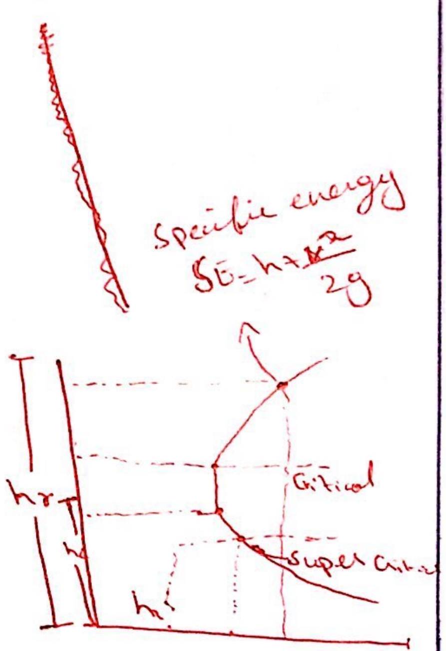


A rectangular channel having bridth "h" and height (h)

Area

Specific energy equation

$$E = h + \frac{v^2}{2g} \text{ (Kinetic energy)} \rightarrow (1)$$



h_1 and h_2 = Alternate height

h_c = critical Depth

As we know that

$$Q = AV$$

or

$$v = \frac{Q}{A}$$

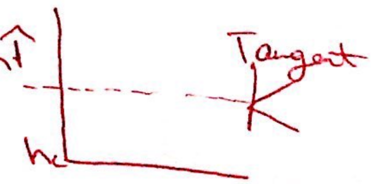
Put $v = \frac{Q}{A}$

we get

$$E = h + \frac{Q^2}{A^2 2g} \quad \text{as } A = b \times h$$

$$E = h + \frac{Q^2}{b^2 h^2 2g} \quad \text{as } q = \frac{Q}{b}$$

$$E = h + \left(\frac{Q}{b}\right)^2 \frac{1}{h^2 \cdot 2g}$$



So

$$E = h + \frac{Q^2}{2g \cdot h^2} \quad (2)$$

derive equation ② w.r.t "h"

$$\frac{dE}{dh} = \frac{d}{dh}\left(hc + \frac{q^2 hc^{3}}{2g}\right)$$

$$\frac{dE}{dh} = \frac{1 + (-2) q^2 hc^{-2-1}}{2g}$$

$$0 = 1 - \frac{\cancel{2} q^2 hc^{-3}}{\cancel{2} g}$$

$$0 = 1 - \frac{q^2}{hc^3 \cdot g}$$

$$0 = \frac{q^2}{hc^3 \cdot g} = 1$$

$$hc^3 \cdot g = q^2$$

$$hc^3 = \frac{q^2}{g}$$

or

$$\boxed{hc = \left(\frac{q^2}{g}\right)^{1/3}}$$ Critical Depth

Now For critical velocity "vc"

As

$$hc^3 = \frac{q^2}{g} \quad -③$$

by C.M

$$hc^3 g = q$$

as $q = \frac{AV}{b}$

$qb = AV$

or

$V = \frac{qb}{A}$ as $A = b \cdot h$

Now

$$\frac{q \times b}{A} = V$$

$$\frac{q \cancel{b}}{\cancel{b} \cdot hc} = V$$

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So

$$v = \frac{q}{h_c}$$

$$q = v h_c$$

From equation ③

$$q^2 = h_c^3 g$$

Put $q = V_c h_c$

or $V_c^2 \cdot h^2 = h_c^3 \cdot g$

$$V_c^2 = \frac{h_c^3 \cdot g}{h_c^2}$$

or $V_c^2 = g \cdot h_c$

$\boxed{V_c = \sqrt{g \cdot h_c}}$ Critical velocity

Therefore

Critical Depth $(h_c) = \left(\frac{q^2}{g}\right)^{1/3}$

Critical velocity "V_c" $= \sqrt{g \cdot h_c}$

Q2.

Given-

Depth of Rectangular channel(d) = ?

Flow rate (Q) = 3.5 m^3/sec

Slope of Bed(S_0) = 0.0008

n = 0.0249

width of bed = 7817 = 7.817

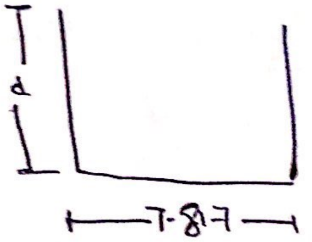
Critical depth = ?

Flow sub critical or super critical = ?

Solution:-

Area = 7.817 × d

= 7.817d



Perimeter = d + 7.817 + d

= 7.817 + 2d

Hydraulic Radius (R_h) = A/P

$$= \frac{7.817d}{7.817+2d}$$

By using Manning Equation

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2}$$

Putting values

$$3.5 = \frac{1}{0.0219} \times 7.817d \times \left(\frac{7.817d}{2d+7.817}\right)^{2/3} \times (0.0008)^{1/2}$$

$d = 0.558m$

$\text{Area} = 7.817(0.558)$

$= 4.36 m^2$

$\text{Perimeter} = 7.817 + 2(0.558)$

$= 8.933 m$

Hydraulic Radius $(R_h) = \frac{4.36}{8.933}$

$= 0.488m$

Finding critical depth:-

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \frac{3.5}{7.817}$$

$= 0.447 m^2/sec$

$$y_{cr} = \left(\frac{(0.447)^2}{9.81}\right)^{1/3}$$

$= 0.273$

As $y > y_{cr}$

$0.558 > 0.273$

So flow is sub-critical

Now critical velocity:-

$V_c = \sqrt{g h_c}$

$V_c = \sqrt{9.81 \times 0.448}$

$\boxed{V_c = 2.096 m/sec}$ critical velocity

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Ans (3)

Given:-

Friction Drag $(F_D) = ?$

width $(B) = 200mm = 0.2m$

Length $(L) = 800mm = 0.8m$

Specific Gravity $(S) = 0.89$

undisturbed velocity $(U) = 5 m/sec$

Kinematic viscosity $(\nu) = 0.93 \times 10^{-4}\ m^2/sec$

Solution:-

Checking whether flow is laminar or not

By Reynold Number

$$R = \frac{DV}{\nu}$$

For smooth flat plate

$D = L\ ,\ V = U$

$$\text{So } R = \frac{LU}{\nu}$$

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}} = 43010$$

$43010 < 500,000 \rightarrow$ laminar

By using Formula:-

$$F_f = C_f \cdot \rho \cdot \frac{V^2}{2} \cdot BL$$

where

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}} = 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} \rightarrow 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890 \, kg/m^3$$

$$F_f = C_f \cdot \rho \cdot \frac{U^2}{2} \cdot BL$$

$$= 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$F_F = 11.39 N$$