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QNO# 01:-

$$f(t) = 1+t, -\pi \leq t \leq \pi.$$

Here we use the formula.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \rightarrow$$

eq (1)

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt.$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt.$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt.$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

P.T.O

$$a_n = \frac{1}{\pi}\left[(1+t)\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right]$$

$$a_n = \frac{1}{\pi}\left((1+t)\cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left[\cos n\pi - \cos n(-\pi)\right]$$

$$a_n = \frac{-1}{n^2\pi}\left[-1-(-1)\right]$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\, dt.$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt \cdot \frac{d}{dt}(1+t)\, dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left[\frac{(1+t)(-\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right]$$

P.T.O

$$b_n = \frac{1}{\pi}\left|-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\cancel{\sin nx}}{n^2}\Big|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cos n\pi + \pi\cos n\pi - \cancel{\cos} n\pi + \pi\cos n(\pi)\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos} n\pi + \pi\cos n\pi - \cancel{\cos} n\pi + \pi\cos n\pi\right)$$

$$b_n = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}.$$

So eqn become

$$\boxed{f(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n}\sin t}$$

QNO#03:- Solve the following System of Linear equations:-

$5x + 4z + 2m = 3$

$x - y + 2z + m = 1$

$4x + y + 2z = 1$

$x + y + z + m = 0$

P.T.O

Solution:-

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4\ R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+4/5 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad -1/5 \times R_3$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad \begin{array}{l} 5 \times R_3\ R_4 \\ 5 \times R_4 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3, 4, 5R_4$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] C_2 \times -5$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 0 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] 5/4 \times R_1$$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \left.\begin{array}{c} 126/84 \\ 31/21 \\ -11/21 \\ 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = \left(3/4, \frac{31}{21}, \frac{-11}{21}, \frac{1}{3}\right)$$

$x = 3/4$

$y = \frac{31}{21}$

$z = \frac{-11}{21}$

$m = 1/3$

QNO#02:-

Ans:-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen Values =?

Sol:-

Step ⇒ 01 :-

we have;

$(A - \lambda I)X = 0$ A= given Matrix

Step = 02

we have; the characteristics equation is given By;

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -4 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

P.T.O

Step # 03

$$\lambda^3 - \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal elem.}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal} \\ \text{minor}\end{matrix}\right| \lambda - |A| = 0 \quad \text{(B)}$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of diagonal ~~elements~~ minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$$= (-6) + (2) + (1)$$

$$= -6 + 2 + 1$$

$$= -3$$

By putting value in eq (B);

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{— (C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$

$$= -6+6$$

$$= 0$$

P.T.O

By putting values in (i) 3

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

using Quadratic formula;

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{array}{l} a = 1 \\ b = -4 \\ c = -3 \end{array}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$$

we have eigen values.

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right) \text{ required } \underline{\text{sol}^{n}}$$

QNO# 04:-

verify that:-

$$u(x,t) = \sin(x+2t)$$

is a Solution of the one — Dimensional wave equations:-

Sol:-

Given that:-

$$u(x,t) = \sin(x+2t)$$

Differentiate w.r.t x partially

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+2t)$$

P.T.O

$$\frac{\partial^2 y}{\partial x^2} = -\sin(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 y}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = -\sin(x+2t)}$$

My $y(x,t) = \sin(x+2t)$

Differentiate W.r.t "t".

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial y}{\partial t} = \cos(x+2t)(0+2)$$

$$\frac{\partial y}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 y}{\partial t^2} = (2) - \sin(x+2t)(0+2)$$

$$\boxed{\frac{\partial^2 y}{d t^2} = -4\sin(x+2t)}$$

As we know that one-dimensional wave equation is.

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

$$-4\sin(x+2t) = c^2\left[-\sin(x+2t)\right]$$

$$-4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

For the orbitary Constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

then it will be verified for the orbitary Constant.

$$c = 2.$$

The End:.