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Section A

Deptt: Civil Engg.

Paper Earthquake

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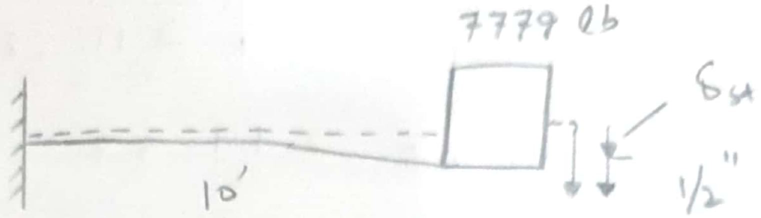
Question No: 01



Given data:

$E = 29000$ ksi

$I = 150$ in^4

$\delta_{st} = 7779$ lb

Solution:

The general EOM for SDOF system:

$$Ku + C\dot{u} + m\ddot{u} = P(t)$$

System is undamped $(c = 0)$

undergoing free vibration $P(t) = 0$

EOM become $ku + m\ddot{u} = 0 \rightarrow (1)$

Now

$$K = \frac{3EI}{L^3} = \frac{3 \times 29000 \times 150 \text{ in}^4}{(10 \times 12)^3}$$

$$K = 7.55 \text{ k/in}$$

$$K = 90625 \text{ lb/ft}$$

$\rightarrow$ $$m = \frac{7779 \text{ lb sec}^2}{32.2 \text{ ft}}$$

$$m = 241.58 \text{ slug}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{241.58}}$$

$$\omega_n = 19.37 \text{ rad/sec}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.37} = 0.324 \text{ sec}$$

→ Substituting the corresponding value in eq ①

$$Ku + m\ddot{u} = 0$$

$$90625\,u + 241.58\,\ddot{u} = 0$$

where 'K' is in lb/ft and 'm' is in lb sec/ft^2

General solution to the EOM for undamped free vibration is:

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$\because\ u(0) = \frac{1}{2} = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24}\ \text{ft}$$

and

$$\dot{u}(0) = 0$$

$$\rightarrow\ u(t) = \left(\frac{1}{24}\right)\left((\cos)\ 19.37\right) + 0 = \left(\frac{1}{24}\right)\left(\cos\right)\ 19.37)\,t$$

Equivalent static force at any time 't' is

$$f_{(s)}(t) = K u(t) = \frac{90625 \times \cos(19.37)}{24}$$

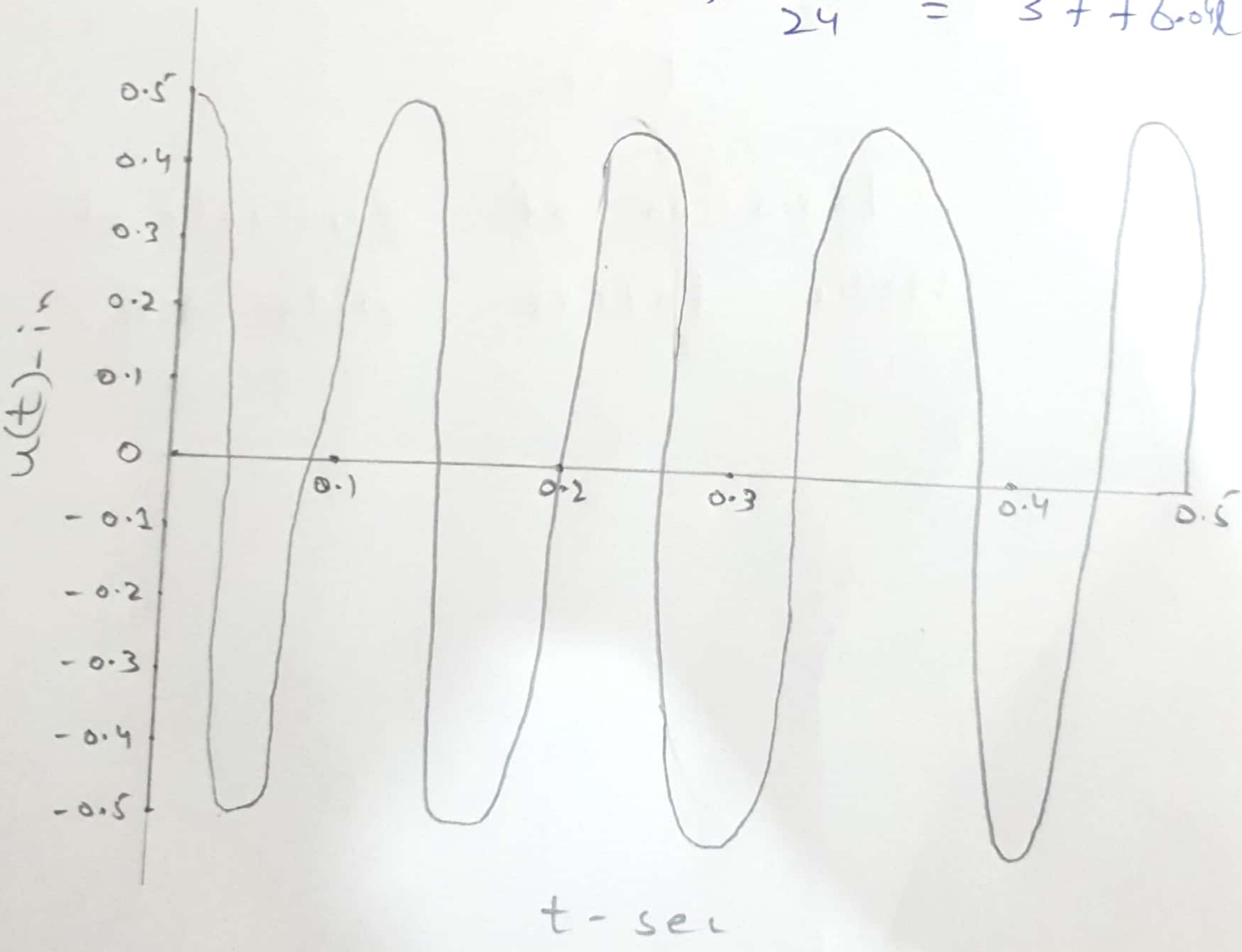
$$f_{(s)}(t) = 3776 \cos(19.37 t)$$

Amplitude of dynamic displacement u_0 for undamped free vibration;

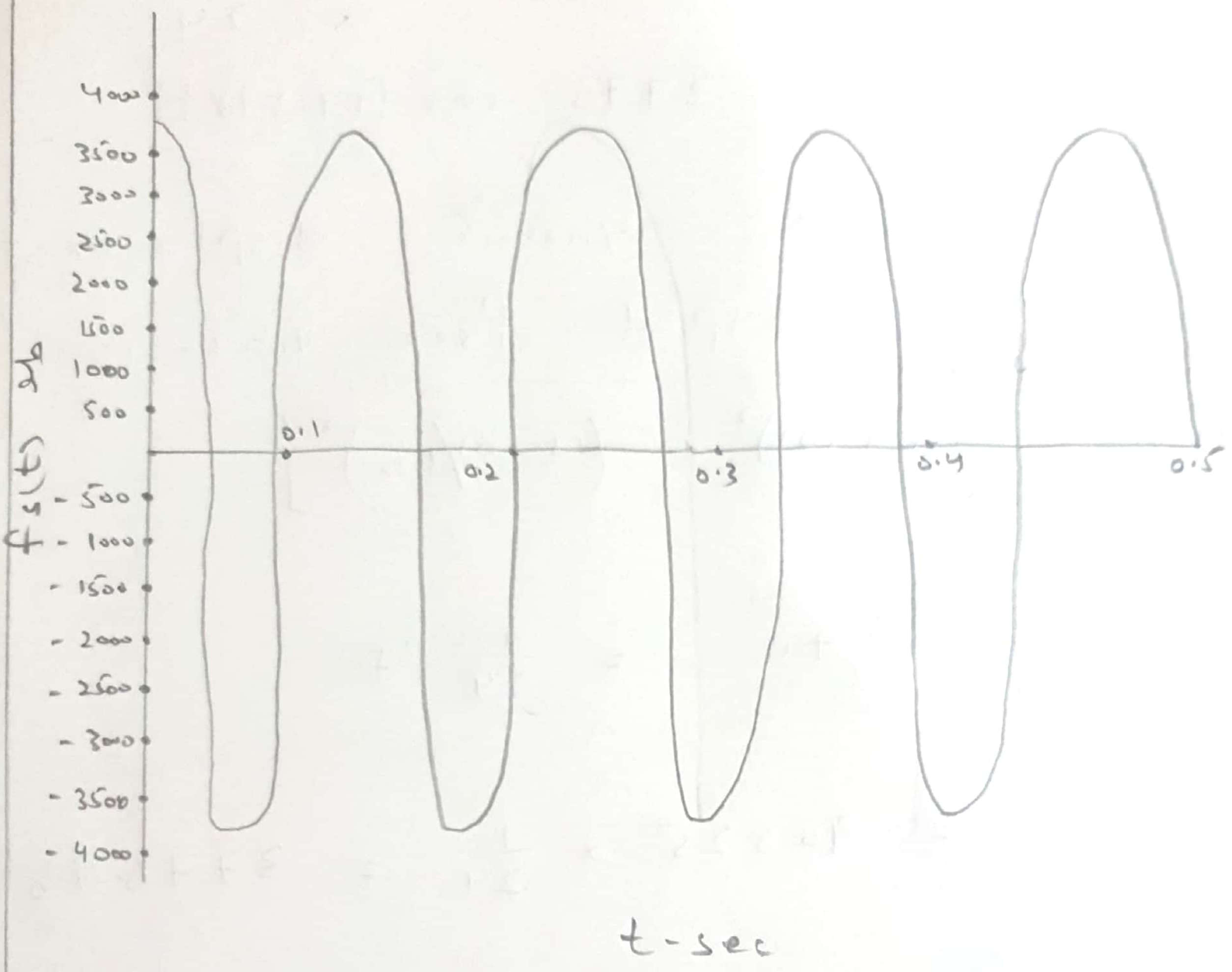
$$u_0 = \sqrt{\left[(u(0))^2 + (\dot{u}(0)/\omega_n)^2\right]}$$

$$u_0 = \sqrt{\left(\frac{1}{24}\right)^2 + 0} = \frac{1}{24} \text{ ft}$$

$$\rightarrow \quad K u_0 = 90625 \times \frac{1}{24} = 3776.0416$$



Variation of displacement with time



Variation of Equivalent static forces with time

Question No: 02

Given data:

Using the data of problem No. 01:

Damping ration of reinforced concrete with considerable cracking = 3 - 5 %

$$\zeta = 3\%$$

Solution:

E.O.M for damped free vibration;

$$ku + c\dot{u} + m\ddot{u} = 0 \rightarrow (1)$$

From question/problem No: 01

$$k = 90625 \text{ lb/ft}$$

$$m = 241.58 \text{ lb.sec}^2\text{/ft}$$

$$\omega_n = 19.37 \text{ rad/sec}$$

$$c = \zeta \times 2m\omega_n$$

$$= 0.03 \times 2(241.58) \times 19.37$$

$$c = 280.76 \text{ lb.sec/ft}$$

By subsitituting values of

K, c and m in eq ①

$$Ku + c\dot{u} + m\ddot{u} = 0$$

$$90625u + 280.76\dot{u} + 241.58\ddot{u} = 0$$

Solution to the E.O.M for damped free vibration is;

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[u(0)\zeta\omega_n\right] \times \sin(\omega_D t)\right]$$

$$\omega_D = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{241.58}}$$

$$\omega_D = 19.37 \text{ rad/sec}$$

$$\rightarrow u(t) = e^{-0.03\times19.37t}\left[\frac{1}{24}\times\cos(19.37t) + \frac{1}{19.37}\times\left[0 + \frac{1}{24}\times0.03\times19.37\right]\times\sin 19.37t\right]$$

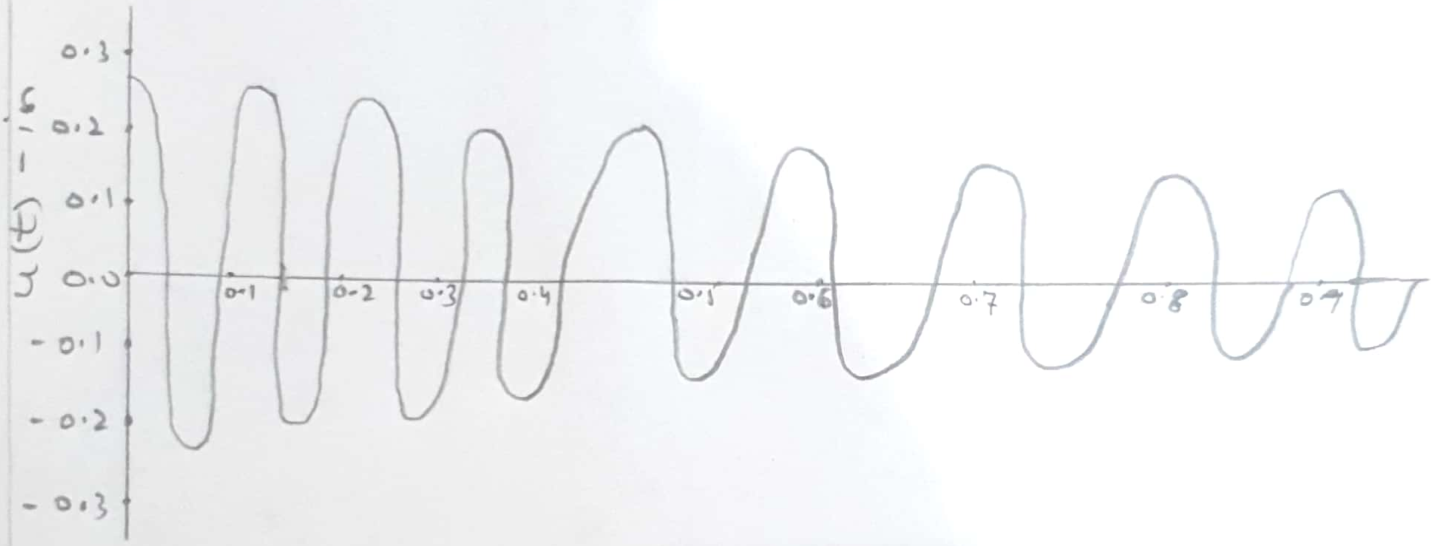
$$\rightarrow u(t) = e^{-0.5811t}\left[0.04166\times\cos(19.37t) + 0.5163 \times 0.0242 \times (\sin 19.37t)\right]$$

$$u(t) = e^{-0.5811t}\left[0.04166\times\cos(19.37t) + 0.01249 \times \sin(19.37t)\right]$$

$$\rightarrow f_s(t) = K \cdot u(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.5811t}\left[(90625 \times 0.04166) \times \cos(19.37t) + (0.01249 \times 90625) \times \sin(19.37t)\right]$$

$$f_s(t) = e^{-0.5811t}\left[3775.46 \times \cos(19.37t) + 1131.9 \times \sin(19.37t)\right]$$



t-sec

Damped free vibration

Damped free vibration

Question No: 03

Given data:

Force = 60 Kips

Displacement of tank = $\left(\frac{ID}{1000}\right)^4$

$= \left(\frac{7779}{1000}\right)$

$= 7.779''$

Time taken to complete 7 cycles = 3.57 sec

Amplitude of displacement

$= 2.286$ cm

$= 0.9''$

Required:

- Damping ratio = ζ = ?
- Natural period of un-damped vibration = ?
- Stifness of structure = ?
- Weight of tank
- Damping cofficient = ?
- Number of cycles to reduce the displacement amplitude to 0.5''.

Solution:

Displacement of tank = $u = 7.779''$

After 7 cycles i.e After $j = 7$

$$u_{j+1} = u_8 = 0.9''$$

Damping ratio:

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{1+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.779}{0.9}\right]$$

$$\zeta(7 \times 2 \times 3.4) = 2.157$$

$$\zeta(43.96) = 2.157$$

$$\zeta = \frac{2.157}{43.96} = 0.0491$$

$$\zeta = 4.91\ \%$$

T_n :

As 7 cycles are completed in "3.57" sec. Thus time required to complete 1 cycle

$$T_D = \frac{3.57}{7} = 0.51 \text{ sec}$$

Now

$$W_D = W_n \sqrt{(1 - \zeta^2)}$$

$$\Rightarrow \frac{2\pi}{W_D} = \frac{2\pi}{W_n} \sqrt{(1 - \zeta)^2}$$

As

$$T_D = T_n / \sqrt{(1 - \zeta^2)}$$

$$\Rightarrow T_n = T_D (1 - \zeta^2)$$

$$= 0.51 \times \left(\sqrt{1 - (0.0491)^2} \right)$$

$$T_n = 0.26 \text{ sec}$$

So the natural period of undamped vibration $= T_n = 0.26$ sec

Stifness of structure = K :

$$K = \frac{F \cos \theta}{7.779}$$

$$= \frac{60 \times \cos 60^\circ}{7.779}$$

$$= 3.86 \text{ k/in}$$

$$K = 46320 \text{ lb/ft}$$

Weight of tank:

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{W/g}} = \sqrt{\frac{K \cdot g}{W}}$$

$$\Rightarrow \quad W_n^2 = \frac{K \times g}{W_n^2}$$

Also $\quad W_n = \frac{2\pi}{T_n}$

$$W = \frac{Kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$W = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46320 \text{ lb}}{\text{ft}} \times \frac{32.2 \text{ ft}}{\text{sec}^2}\right] \times \frac{(0.51 \text{ sec})^2}{4\pi^2}$$

$$W = 9826.64 \text{ lb}$$

$$= 9.83 \text{ K}$$

Damping cofficient $= c = ?$

It's known that

$$\zeta = \frac{c}{2m W_n}$$

$$c = \zeta(2m W_n) = \zeta\left(2m\left(\frac{2\pi}{T_n}\right)\right)$$

By putting values

$$c = \frac{0.0491 \times 4 \times \pi \times \left(\frac{9826.64}{32.2}\right)}{0.26}$$

$$c = 724.2 \ lb \cdot sec/ft$$

Number of cycles to reduce the displacement amplitude to 0.5", j

$$j = \frac{1}{2\pi \zeta} \ln \left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.0491} \ln \left[\frac{7.779}{0.5}\right]$$

$$j = 8.896 \text{ or } 9 \text{ cycles}$$

~~End~~