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Subject Linear Algebra

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Section A

Program BS(SE)

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# QUESTION 1

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\text{-}last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 10 \end{bmatrix}$$

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Solution

$$\begin{bmatrix} 1 & 5 & 3 & 0 & 5 \\ 0 & 1 & 4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

$$x_1 + 5x_2 + 3x_3 = 5$$
$$x_2 + 4x_3 = 7$$
$$x_3 = -6$$
$$x_4 = 5$$

$-4R_3 + R_2$

$$x_2 + \cancel{4x_3} = 7$$
$$\underline{-\cancel{4x_3} = 24}$$
$$x_2 = 31$$

So The equation will be

$$x_1 + 5x_2 + 3x_3 = 5$$
$$x_2 = 31$$
$$x_3 = -6$$
$$x_4 = 5$$

$3R_3 - R_1$

$$x_1 + 5x^2 + 3x_3 = 5$$
$$\underset{+}{-} 3x_3 = \underset{-}{+} 18$$

$$x_1 + 5x_2 = 23$$

$5R_2 - R_1$

$$x_1 + \cancel{5x_2} = 23$$
$$\underset{+}{-} \cancel{5x_2} = \underset{+}{-} 155$$

$$x_1 = -132$$

So the values are

$x_1 = -132$
$x_2 = 31$
$x_3 = -6$
$x_4 = 5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -132 \\ 0 & 1 & 0 & 0 & 31 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

Verfication taking first equation

$$x_1 + 5x_2 + 3x_2 = 5$$
$$-132 + 5(31) + 3(-6) = 5$$
$$-132 + 155 - 18 = -5$$

$$-150 + 155 = 5$$
$$5 = 5$$

Hence Matrix argument matrix

# Question No. 2

Find the elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

In order to transforms first matrix into the Second we will multiply -2 with row 2 and then add the row 2 to row 3

$-2R_2 + R_3$ :·

$$-2 \times \text{row } 2 \quad 0 \; 1 \; -4 \; 2$$
$$= 0 \; -2 \; 8 \; -4$$

Now adding it to row 3

$$\begin{array}{rrrrr} + & 0 & -2 & 8 & -4 \\ & 0 & 2 & -5 & -1 \\ \hline & 0 & 0 & 3 & -5 \end{array}$$

Putting the newly formed row 3 into the first matrix.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Hence we transformed the first matrix into the Second matrix.

Now adding reverse row operation and turning the second matrix into the first

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Multiplying 2 with row 2 and adding it to row 3

$2R_2 + R_3$:-

$$\begin{array}{cccc} 0 & 2 & -8 & 4 \\ 0 & 0 & 3 & -5 \\ \hline 0 & 2 & -5 & -1 \end{array}$$

Putting back the new row 3 obtained above, into the second matrix and transforing it to matrix no first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}.$$

# QUESTION NO. 2

## PART B

Given below are some matrices. find whether these are in the forms written infront of them or not. Explain in your own words of each of the selection in detail

a
$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & \pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

This matrix is not in echelon form because the most below row have "e" element which is not zero and for echelon from there must be all zero element in the last line.

b
$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b) This one is the echelon form because the pivot element having zero in their below row and the last row have also zero element.

e $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

c) This is not reduced echelon form because having non-zero rows at the bottom.

) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

This is reduced echelon form if when we transpose $R_2 \longleftrightarrow R_3$ then the bottom line will be have all zero element.

# QUESTION 3

## PART "A"

(a)
The row echelon form is used to solve the System of linear equations what is difference between the row echelon and reduced row echelon form?. What is the practical use of reduced row echelon form? Give an one example?

**Row Echelon Form:** A Matrix is said to be in row (coloumn) echelon for when it Satisfies the following Conditions.

1- The first non-zero element in each row, called leading entry is 1.

2 Each leading entry is in a row to the right of the leading entry in the previous row.

3 Rows with all zero elements, if any are below (after) the row havin

a non-zero element

For Example:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Reduced Row Echelon form:

* A Matrix is said to be in reduced row echelon form when it satisfies the following conditions

1- The Matrix satisfies conditions for a row echelon form

2- The leading entry in each row is the only non-zero entry in its row.

For example

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore we can say that each reduced row echelon form is also a row echelon form, but vice versa is not always true

# Question No. 3

## Part "B"

$$\begin{bmatrix} 1 & 1D_2 & 8 \\ 2 & 8 & -1 \\ -1D3 & 0 & 0 \\ 1 & -4 & 10\text{-first-last} \end{bmatrix}$$

Solution :

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -5 & 0 & 0 \\ 1 & -4 & 14 \end{bmatrix} \quad R_3 + 5R_4$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & 0 \\ 1 & 4 & 14 \end{bmatrix} \quad R_1 \longleftrightarrow R_4$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 1 & -4 & 14 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 - 2R_3$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 0 & -29 \\ 1 & -4 & 14 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 0 & -29 \\ 0 & -10 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$ $R_3 - 1R_1$

$R_3 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -10 & 6 \\ 0 & 0 & -29 \\ 0 & 0 & 0 \end{bmatrix}$$ Ans.