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SUBJECT # Numerical Analysis

SECTION # "B"

SEMESTER # "4th"

DATE # 19/Aug/20

Q1- Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} - 0.165x^2$$

Use newton raphson Method with

$$x_0 = 0.02$$

Sol Rearranging the equation.

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

$$= 0.09 - \frac{0.00033}{-0.0054}$$

$$\boxed{x_1 = 0.081}$$

$$\rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x_1) = -0.00016$

$f'(x_1) = -0.0070$

$$= 0.081 - \frac{(-0.00016)}{-0.0070}$$

$$\boxed{x_2 = 0.058}$$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$f(x_2) = 0.000030$

$f'(x_2) = 0.0090$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$$\boxed{x_3 = 0.061}$$

$Q_2$ :- Use the numbers $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the Lagrange interpolation polynomial for $f(x) = 1/x$ at $x = 3$.

Sol :- putting the values in the function

$$y_0 = 0.5, \quad y_1 = 0.364, \quad y_2 = 0.25$$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)y_2}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.364$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\Rightarrow y = 0.33$$

Q3:- Complete the divided difference table for the given and Construct the interpolating polynomial that uses all this data.

| | | | | | |
|---|---|---|---|---|---|
| $x =$ | 1.0 | 1.3 | 1.6 | 1.9 | 2.7 |
| $Y =$ | 0.7651977 | 0.6200860 | 0.4554022 | 0.2818186 | 0.1103623 |

| $x_i$ | $f(x_i)$ | $f(x_{i-1}, x_1)$ | $f(x_{i-2}, x_{i-1}, x_1)$ | $f(x_{i-3}, x_1)$ | $f(x_1, \ldots, x_L)$ |
|---|---|---|---|---|---|
| $x_0$ 1 | 0.7651977 | | | | |
| | | -0.4837056 | | | |
| $x_1$ 1.3 | 0.6200860 | | -0.108734 | | |
| | | -0.548946 | | 0.0658785 | |
| $x_2$ 1.6 | 0.4554022 | | -0.0494433 | | -0.0028049. |
| | | -0.578692 | | 0.0625125 | |
| $x_3$ 1.9 | 0.2818186 | | | | |
| | | 0.571521 | 0.006818 | | |
| $x_4$ 2.7 | 0.1103623 | | | | |

i) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = -0.4837056}$

2) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = -0.548946}$

3) $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$

4) $$f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$$

$$\boxed{f(x_3, x_4) = 0.571521}$$

## Second divided difference.

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{0.548946 - (-0.4837056)}{1.6 - 1}$$

$$\boxed{= -0.108734}$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (0.548946)}{1.9 - 1.3}$$

$$\boxed{f(x_1, x_2, x_3) = -0.0494433}$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

## Third divided difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= 0.0658785$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$= 0.0625255$$

## 4th divided difference.

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.0625255 - 0.0658785}{2.2 - 1}$$

$$= 0.0028049$$

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2)$$
$$+ (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)$$
$$(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4).$$

$$= 0.7651977 + (x-1) - 0.4837056 + (x-1)(x-1.3)(-0.108734)$$
$$+ (x-1)(x-1.3)(x-1.6)(0.0658785) + (x-1)(x-1.3)$$
$$(x-1.6)(x-1.9)(-0.0028049).$$