Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	SIR Pir Meher Ali Shah	**Total Marks:**	50

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation [illegible] [illegible]	Marks 7 CLO 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. [illegible]	Marks 7 CLO 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform [illegible] (Hint: Take inverse z-transform using partial fraction method)	Marks 6 CLO 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral [illegible]	Marks 6 CLO 2
Q.3	(a)	A two- pole low pass filter has the system response [illegible] Determine the values of b_0 and p such that the frequency response $H(\omega)$ satisfies the [illegible]	Marks 6 CLO 3
	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in at $\omega = 4\pi/9$.	Marks 6 CLO 3
Q 4	(a)	A finite duration sequence of Length L is given as [illegible] Determine the N- point DFT of this sequence for $N \geq L$	Marks 6 CLO 2
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step [illegible]	Marks 6 CLO 2

Ans ① ⓐ $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$x(n) = (-1)^n u(n)$

initial conditions are;

$y(-1) = y(-2) = 0$

Solution:-

First we determine the solution to the homogeneous equation. we assume the solution to be exponential

$y_h(n) = \lambda^n$

The characteristic equation will be;

$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2}$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$

Hence

$y_h(n) = C_1 2^n + C_2 n 2^n$

The particular solution is;

$y_p(n) = K(-1)^n u(n)$

Substituting this solution into the difference equation, we obtain.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$

$$K(1+4+4) = 2$$

$$K = \frac{2}{9}$$

So the total Solution is

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

And from the initial conditions, we obtain $y(0) = 1$, $y(1) = 2$ then;

$$c_1 + \frac{2}{9} = 1$$

$$c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$c_2 = \frac{1}{3}$$

So $y(n)$ will be;

$$y(n) = \left[\frac{7}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

Ans (1) (b) $y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$

Solution:-

the characteristic equation is ;

$\lambda^2 - 0.7\lambda + 0.1 = 0$

hence;

$\lambda = \frac{1}{2}, \frac{1}{5}$

So $x(n) = \delta(n)$

$y(0) = 2$

$y(1) - 0.7y(0) = 0$

$y(1) = 1.4$

Hence $c_1 + c_2 = 2$

and $\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$

$c_1 + \frac{2}{5}c_2 = \frac{14}{5}$

So these equations yield

$c_1 = \frac{10}{3}, \quad c_2 = \frac{4}{3}$

$u(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$

$h(n) = 30$

Now the step response is;

$$\delta(n) = \sum_{K=0}^{n} h(n-K)$$

$$= \frac{10}{3} \sum_{K=0}^{n} \left(\frac{1}{2}\right)^{n-K} - \frac{4}{3} \sum_{K=0}^{n} \left(\frac{1}{5}\right)^{n-K}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{K=0}^{n} 2^{K} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{K=0}^{n} 5^{K}$$

$$\delta n = \frac{10}{3} \left(\frac{1}{2}\right)^{n} (2^{n+1} - 1) u(n) -$$

$$\frac{1}{3} \left(\frac{1}{5}\right)^{n} (5^{n+1} - 1) u(n)$$

So this is the step response.

Ans (2) (a) $x(z) = \dfrac{1}{(1-2z^{-1})(1-z^{-1})^2}$

Solution:-

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

By partial fractional method

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$= \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1}) \quad \text{— (A)}$$

Now put $z = 1$ in equ (A)

$$1 = A(1-1)^2 + B(1-2)(1-1) + C(1)(1-2)$$

$$1 = 0 + 0 - C$$

$$\{C = -1\}$$

Now put $z = 2$ in equ (A)

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$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) +$$

$$C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(0)$$

$$1 = \frac{A}{4} + 0 + 0$$

$\{A = 4\}$

Now put $z = 3$ in eqn (A)

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$$

$$1 = \frac{4A}{9} + \frac{2B}{9} + \frac{C}{9}$$

put value of A and C

$$1 = \frac{4\times4}{9} + \frac{2B}{9} + \frac{(-1)}{9}$$

$$1 = \frac{16}{9} + \frac{2B}{9} - \frac{1}{9}$$

$$1 + \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$\frac{9+1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$\frac{10}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$\frac{2}{9}B = \frac{10-16}{9}$$

$$\frac{2}{9}B = \frac{-6}{9}$$

$$B = \frac{-\cancel{6}^{3}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{2}_{1}}$$

$$\{B = -3\}$$

Hence;

$$x(n) = [4(2)^n - 3 - n]u(n)$$

Ans (2)(b) $X(Z) = \frac{1}{1-aZ^{-1}}$ $|Z| > |a|$

Solution:-

Using the complex inversion method;

we have;

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{Z^{n-1}}{1-aZ^{-1}} dZ$$

$$= \frac{1}{2\pi j} \oint \frac{Z^n dZ}{Z-a}$$

Here C is a circle radius greater than $|a|$.

$$\frac{1}{2\pi j} \oint_c \frac{f(Z)}{Z-Z_0} dZ = \begin{cases} f(Z_0) & \text{if } Z_0 \text{ is inside } C \\ 0 & \text{if } Z_0 \text{ is outside } C \end{cases}$$

here $f(Z) \approx Z^n$

So we distinguish Two cases.

(i) If $n \geq 0$, $f(Z)$ has only Zeros and hence no poles inside C. The only pole inside C is $Z = a$, hence

$$x(n) = f(Z_0) = a^n$$

where $n \geq 0$

(2) if $n<0$, $f(z)=z^n$ has an nth-order pole at $z=0$, which is also inside C. Thus there are contributions from both poles.

For $n=-1$ we have

$$x(-1)=\frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)}\,dz$$

$$=\left.\frac{1}{z-a}\right|_{z=0}+\left.\frac{1}{z}\right|_{z=a}\approx 0$$

if $n=-2$ we have;

$$x(-2)=\frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}\,dz$$

$$=\left.\frac{d}{dz}\left(\frac{1}{z-a}\right)\right|_{z=0}+\left.\frac{1}{z^2}\right|_{z=a}=0$$

By continuing the same way we can show that $x(n)=0$ for $n<0$. Thus

$$x(n)=a^n u(n)$$

Ans (3) (a) $H(z) = \dfrac{b_0}{(1 - P z^{-1})^2}$

$H(0) = 1$

$H(\frac{\pi}{4}) = \frac{1}{2}$

Find b_0 and P

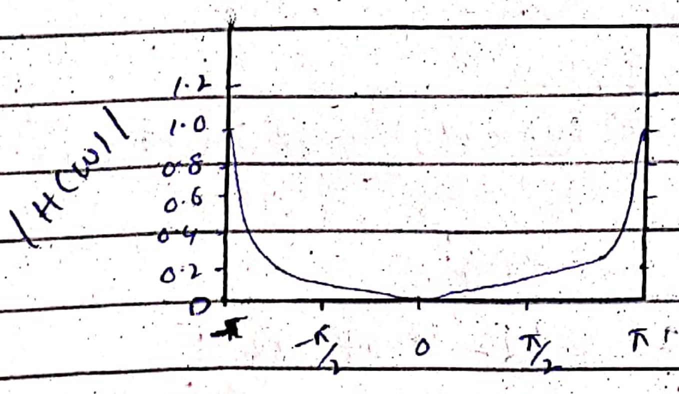
Solution:-

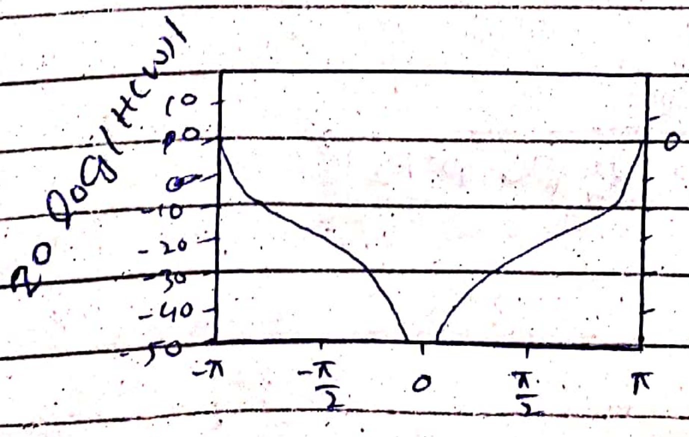
At $\omega = 0$

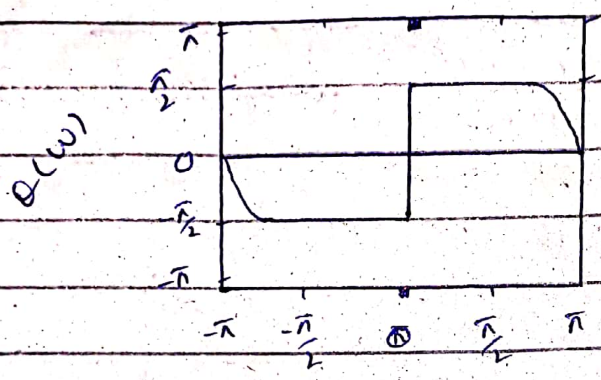
we have

$$H(0) = \frac{b_0}{(1-P)^2}$$

So $b_0 = (1-P)^2$







Magnitude and phase response of a simple highpass filter;

$H(z) = [(1-a)/2][(1-z^{-1})/(1+az^{-1})]$

with $a = 0.9$

At $\omega = \frac{\pi}{4}$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence

$$= \frac{(1-P)^4}{[(1-P/\sqrt{2})^2 + P^2/2]^2}$$

$$= \frac{1}{2}$$

or equivalently;

$$\sqrt{2}(1-p)^2 = (1+p)^2 - \sqrt{2}p$$

The value of $p = 0.32$ satisfies the equation.

The system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same principles can be applied for the design of bandpass filter.

Ans (3) (b) $\omega = \pi/2$

Zero in its frequency response characteristics at $\omega = 0$. $\omega = \pi$ and magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = \frac{4\pi}{9}$

Solution:-

The filter must have poles at

$$P_{1,2} = re^{\pm j(\pi/2)}$$

and zeros at $z = 1$ and $z = -1$

So the system function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2 - 1}{z^2 + r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$

So we have;

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(w)$ at $w = 4\pi/9$.

Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\ \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

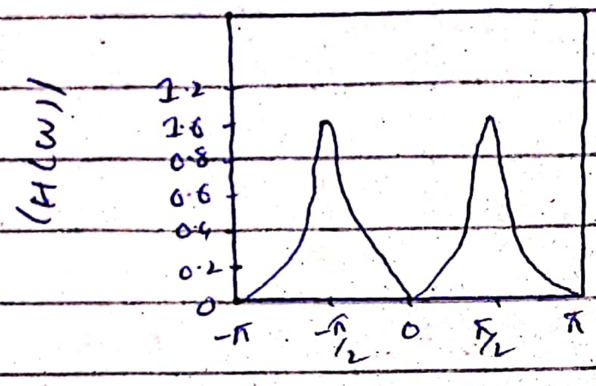
$$= \frac{1}{2}$$

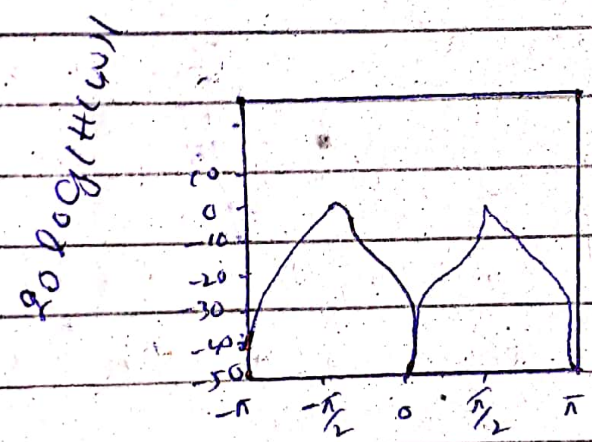
or

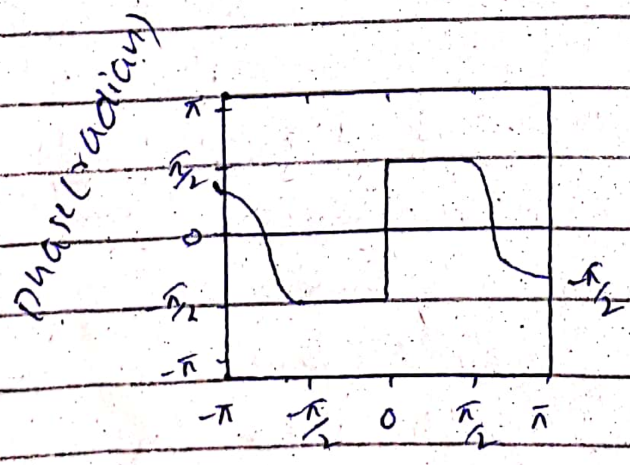
$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. So the system function for the desired filter is;

$$H(z) = 0.15\frac{1-z^{-2}}{1+0.7z^{-2}}$$







Magnitude and phase response of a simple band pass filter.

$H(z) = 0.15[(1 - z^{-2}) / (1 + 0.7z^{-2})]$

Ans (4) (a) $x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$

Determine the N-point DFT of this sequence for $N \ge L$

Solution:

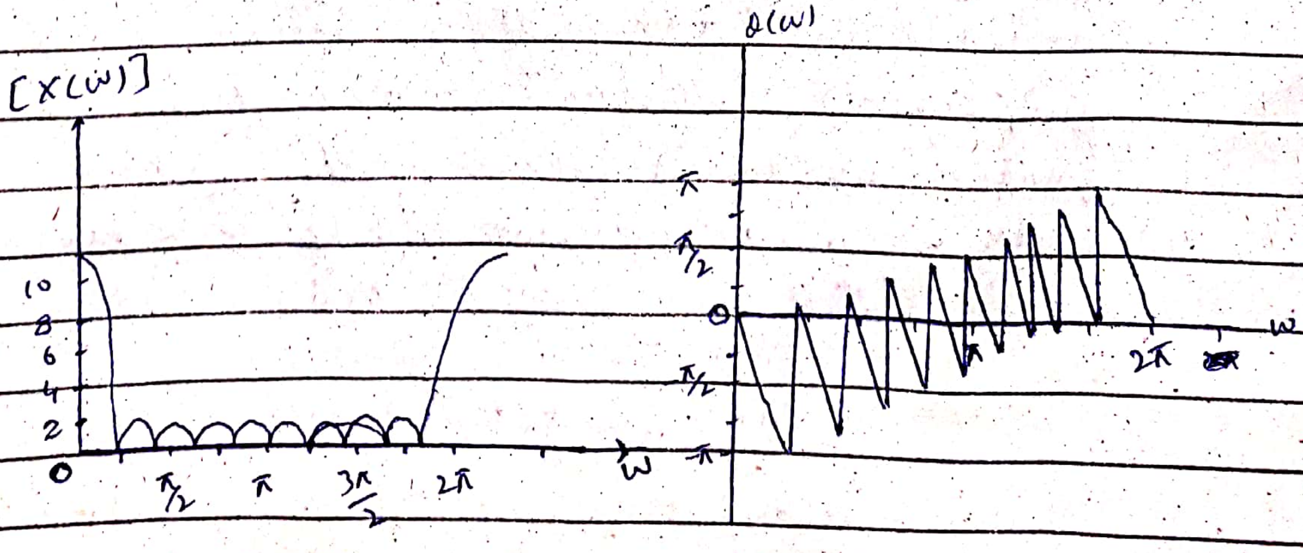
The fourier transform of the sequence is;

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n}$$

$$= \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} \approx \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ is given below for $L=10$.



$\omega_k = 2\pi k / N$

$k = 0, 1 \ldots\ldots N-1$

Hence

$$X(k) = \frac{1 - e^{-j2\pi k L/N}}{1 - e^{-j2\pi k/N}} \quad ; k = 0, 1 \ldots\ldots N-1$$

$$= \frac{\sin(\pi k L/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

if N is selected such that $N = L$ then the DFT becomes;

$$X(k) = \begin{cases} L & k = 0 \\ 0, & k = 1, 2, \ldots\ldots L-1 \end{cases}$$

There is only one non-zero value in the DFT. This is apparent from observation of $X(\omega)$ since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi k/L$, $k \neq 0$.

Ans ④ ⑤ Circular convolution

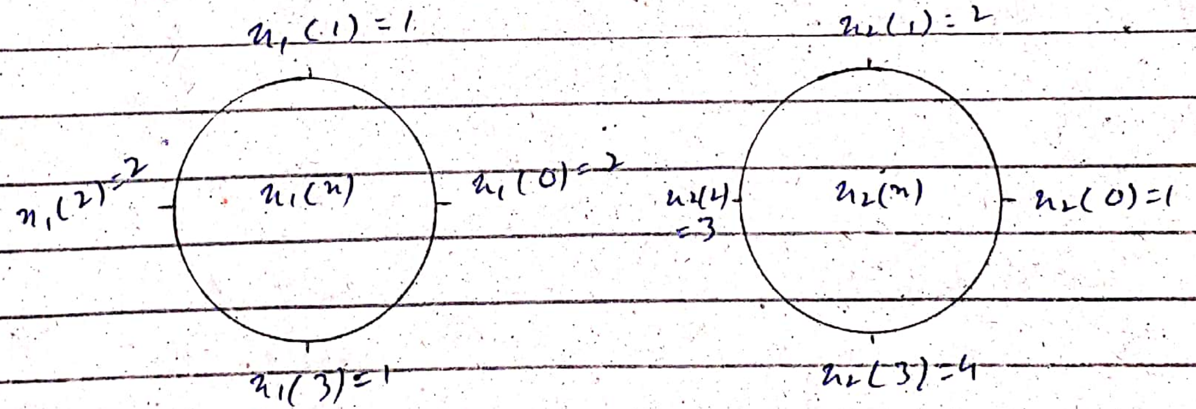
$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Solution:-

Each Sequence consist of four non-zero points.

The Sequence $x_1(n)$ and $x_2(n)$ are graphed below.



Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified by

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N$$

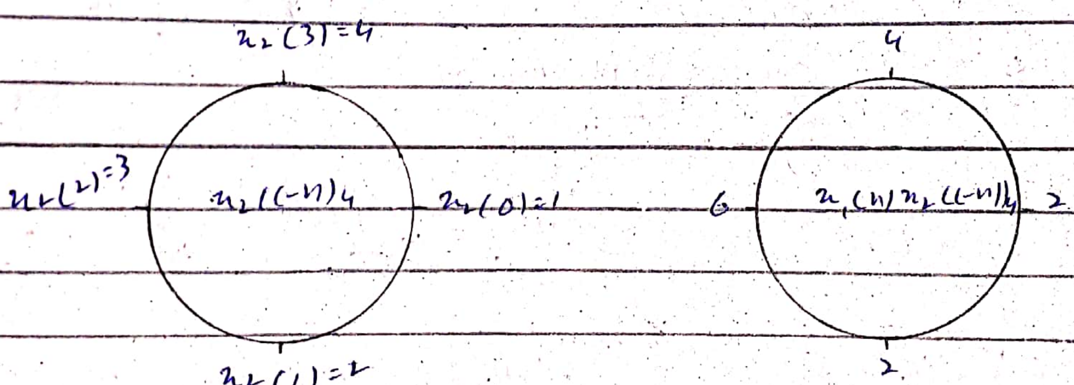
$$\because m = 0, 1, \ldots, N-1$$

Begining with $m=0$

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$ point by point.



Folded Sequence

Product Sequence.

Finally we sum the values in the product sequence to obtain $x_3(0) = 14$

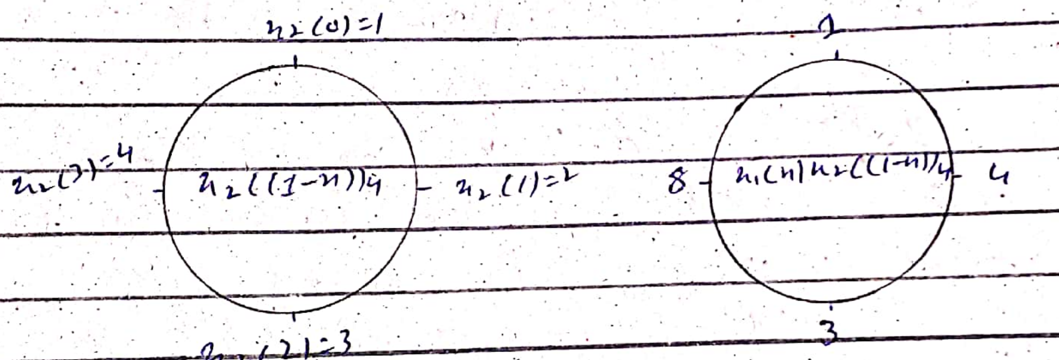
For $m=1$

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

$x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counter clockwise by one unit.

The rotated sequence multiplies $x_1(n)$ to yield the product sequence. Finally we sum the values in the product sequence to obtain $x_3(1)$.

$$x_3(1) = 16$$

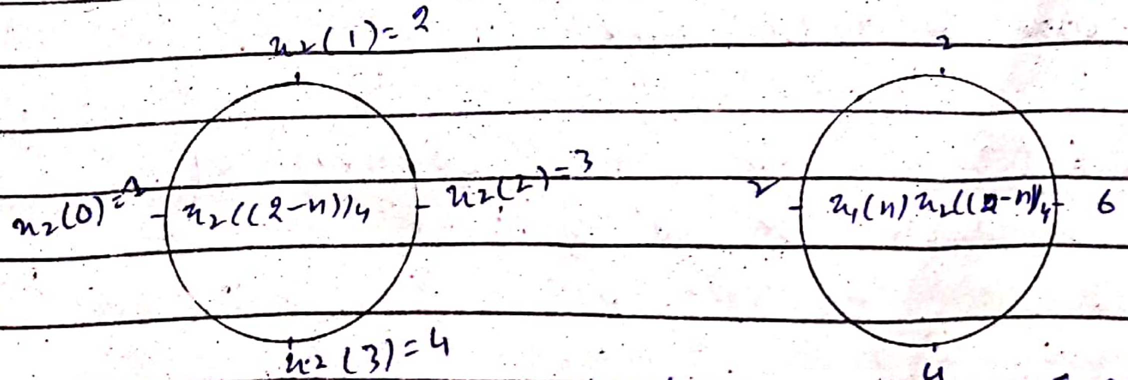


Folded sequence rotated by one unit in time   Product sequence

For $m=2$

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-4))_4$$

Now $x_2((2-n))_4$ is the folded sequence rotated two units of time in the counterclockwise direction.



Folded sequence rotating by two unit   Product sequence

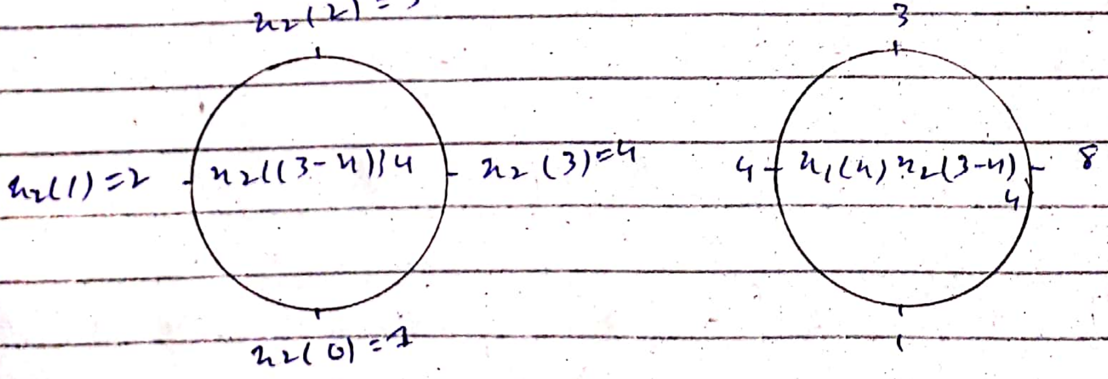
Along with the product sequence $x_1(n)\, x_2((2-n))_4$. By summing we get:

$$x_3(2) = 14$$

For $m=3$

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

The folded sequence $x_2((-n))_4$ is now rotated by three units in time to yield $x_2((3-n))_4$ and resultant sequence is multiplied by $x_1(0)$ to yield the product sequence.



Folded sequence rotated by three units in time.

Product sequence

The sum of the values of product sequence is

$$x_3(3) = 16$$

we observe that if the computation above is continued beyond $m=3$.

There fore the circular convolation of the sequence $x_1(n)$ and $x_2(n)$ yields the sequence

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

we can say that either one of the two sequences may be folded and rotated without changing of the result of the circular convolution. So;

$$x_3(m) = \sum_{n=0}^{N-1} x_2(n)\, x_1((m-n))_N$$

And $m = 0, 1, \ldots, N-1$