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Mid Term.

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Sec : "B"

Subject : Numerical Analysis.

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Q No: 1

Find a root of the equation  $x^3 + 3.993 \times 10^4 = 0.165x^2$

Use Newton Raphson

method with  $x_0 = 0.02$ .

Sol:-

Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad \left. \begin{array}{l} f(x_0) = 0.00033 \\ f'(x_0) = -0.054 \end{array} \right\}$$
$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$x_1 = 0.081$$

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$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.081 - \frac{(-0.00016)}{-0.0070}$$

$$x_2 = 0.058$$

$$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$$x_3 = 0.061$$



Q3:

Complete

data.

$x_i$	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, x_{i-1}, x_i)$	$f(x_{i-4}, x_{i-1}, x_i)$
$x_0$					
$x_1$	0.7651977				
$x_2$	0.6200860	-0.4837056			
$x_3$	0.4554022	-0.548946	-0.108734		0.0658785
$x_4$	0.281886	-0.57862	-0.0494433		-0.002849
$x_5$	0.1103623	-0.571521	0.006818		0.0625255

$$i) f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \frac{0.6200860 - 0.7651977}{13.1}$$

$$f(x_0, x_1) = -0.4837056$$

$$2) f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{0.4554022 - 0.6200860}{1.6 - 1.3}$$

$$f(x_1, x_2) = -0.548946$$

$$3) f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

$$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$$

$$f(x_2, x_3) = 0.578612$$

$$4) f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$$

$$f(x_3, x_4) = -0.571521$$

## Second divided difference

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

### Third divided difference

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= 0.0658785$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$= 0.06251255$$

### 4<sup>th</sup> divided difference.

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= 0.0028049$$



(9)

$$f(x) = f(x_0) + (x-x_0) f'(x_0, x_1) + (x-x_0)(x-x_1) f''(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f'''(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3) f^{(4)}(x_0, x_1, x_2, x_3, x_4)$$

$$= 0.7651977 + (x-1) (-0.4837056) + (x-1)(x-1.3) (0.108734) + (x-1)(x-1.3)(x-1.6) (0.0658785) + (x-1)(x-1.3)(x-1.6)(x-1.9) (-0.0028049)$$

