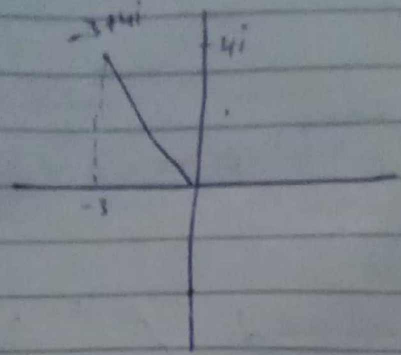
1

Abdul Aziz 13741

Q1:(a)



Polar form and graphically represented $-3+4i$.

(b) A complex function $w=f(z)$ is said to be analytic at a point z_0. if f is differentiable at z_0 and every point in some neighborhood of z_0.

$f(z) = z^2 + z$ for z is analytic

and $f(z) = x^2 - y^2 + x + i(2xy + y)$.

Thus $u = x^2 - y^2 + x$, $v = 2xy + y$.

We can say that

$$\frac{\partial u}{\partial x} = 2x + 1 = \frac{\partial v}{\partial y} \quad \Big| \quad \frac{\partial u}{\partial y} = -2y = -\frac{\partial v}{\partial x}$$

Q2

$$z_1 z_2 = (5+3i)(4-2i)$$

$$= 20 - 10i + 12i - 6i^2$$

$$= 20 + 2i + 6$$

$$=$$

$$z_1 z_2 = 2i + 26$$

$$\frac{z_1}{z_2} = \frac{5+3i}{4-2i} \times \frac{4+2i}{4+2i}$$

$$= \frac{20 + 22i - 6}{4^2 - 4i^2}$$

$$= \frac{22i + 14}{16 + 4}$$

$$= \frac{22i + 14}{20}$$

$$= \frac{11i + 7}{10}$$

$$= \frac{11}{10} i + 7/10$$

Q4 ii

$$\frac{d}{dz} f(z) = \frac{d}{dz}\left(3z^4 - 5z^3 + 2z + 1\right)$$

$$\frac{d}{dz} f(z) = 12z^3 - 15z^2 + 2$$

i

$$\frac{d}{dz} f(z) = \frac{d}{dz}\left(\frac{z^2}{5z+2}\right)$$

$$= \frac{(5z+2)(2z) - z^2(5)}{(5z+2)^2}$$

$$= \frac{10z^2 + 4z - 5z^2}{(5z+2)(5z+2)}$$

$$= \frac{5z^2 + 4z}{(5z+2)(5z+2)}$$

Q3

$u(x,y) = x^3 - 3xy^2 - 5y$

A function is harmonic if

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

$u = x^3 - 3xy^2 - 5y$

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$

$\frac{\partial^2 u}{\partial x^2} = 6x$

Now $\frac{\partial^2 u}{\partial y} = 0 - 6xy - 5$

$\frac{\partial^2 u}{\partial y} = -6x$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$6x - 6x = 0$

Thus it's haromic.

Conjugate of that function.

$u_x = 3x^2 - 3y^2$

$u_y = -6xy - 5$

$u_x = u_y$

$\int 3x^2 - 3y^2 = \int -6xy - 5$

$\cancel{3x^2 - 3y^2 + 6xy + 5}$

$3x^2 + 6xy = 3y^2 - 5 + \cancel{v(x,y)})$

$x^3 + 3x^2y + 5x = y^3 +$
$+ K(x)$

$K(x) = y^3 - x^3 + 3x^2y + 5x$

Conjugate