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Section : A

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Question: 1

Given data:

The state of stresses which are given as;

$\sigma x = -40MPa$

$\sigma y = 70MPa$

$\tau xy = -50MPa$

Required data:

Determine the state of stress at point rotated at 15° in clockwise direction

Solution:

The given stresses are

$\sigma x = -40MPa$

$\sigma y = 70MPa$

$\tau xy = -50MPa$

when we rotate the element of beam at 15° in clockwise direction then we find σx_1, σy_1 and $\tau x_1 y_1$

First we find σx_1 As,

$$\sigma x_1 = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta + \tau xy \sin 2\theta$$

So

$$\sigma_{x_1} = \frac{-40+70}{2} + \left(\frac{-40-70}{2}\right)\cos 2(-15^\circ) + (-50)\sin 2(-15^\circ)$$

$$\boxed{\sigma_{x_1} = -7.63\ \text{MPa}}$$

Now we have to find σ_{y_1}

$$\sigma_{y_1} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{y_1} = \frac{-40+70}{2} - \left(\frac{-40-70}{2}\right)\cos 2(-15) - (-50)\sin 2(-15)$$

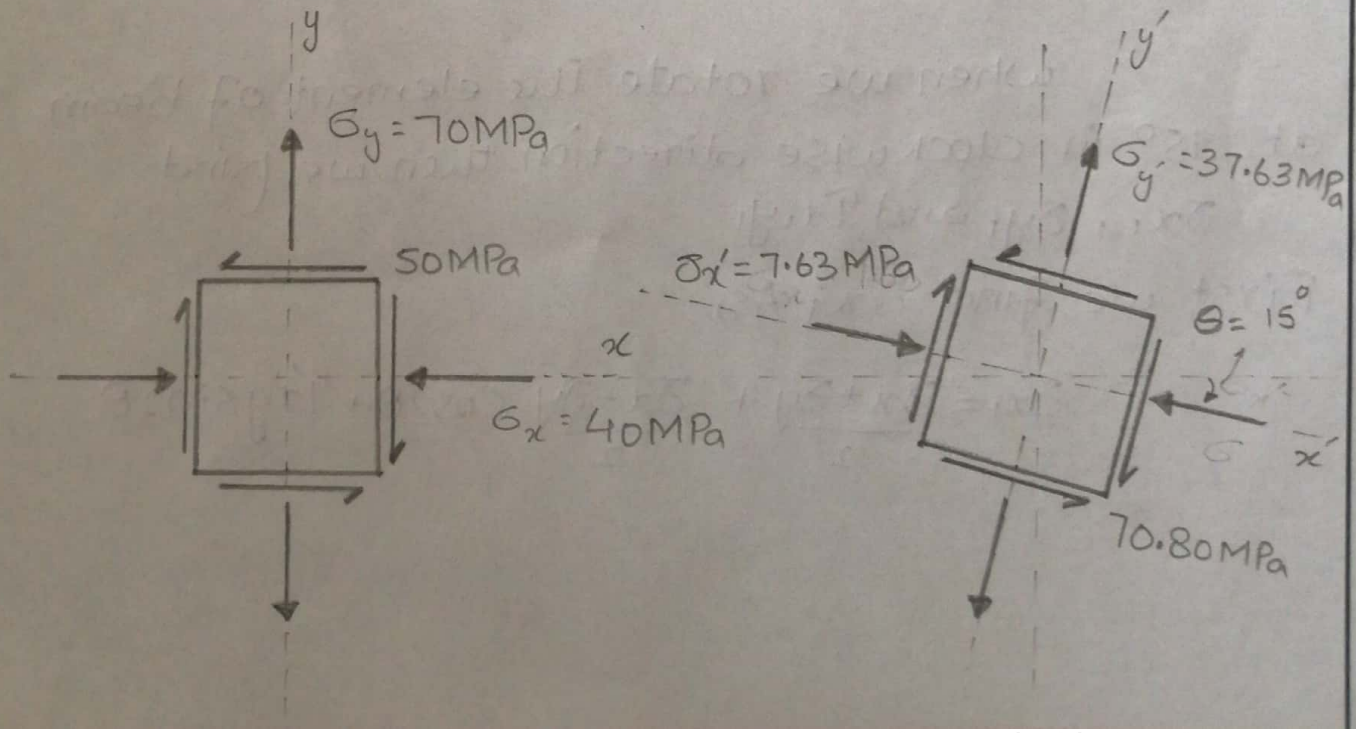
$$\boxed{\sigma_{y_1} = 37.63\ \text{MPa}}$$

Also we have to find $\tau_{x_1y_1}$

$$\tau_{x_1y_1} = -\frac{(\sigma_x-\sigma_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x_1y_1} = -\frac{(-40-70)}{2}\sin 2(-15^\circ) + (-50)\cos 2(-15)$$

$$\boxed{\tau_{x_1y_1} = -70.80\ \text{MPa}}$$



Question : 2

Part : a :

Given data :

The state of stresses which are given as;

$\sigma_x = -40 MPa$

$\sigma_y = 70 MPa$

$\tau_{xy} = -50 MPa.$

Required data :

Principal plane = ?

Principal stresses = ?

Solution :

First we find principle plane;

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2(-50)}{-40-70}$$

$$\tan 2\theta_p = 0.9091$$

$$2\theta_p = \tan^{-1}(0.9091) = 42.27$$

$$\theta_{P_2} = \frac{42.27}{2}$$

$$\boxed{\theta_{P_2} = 21.14^\circ}$$ For X-Axis

Also

$$2\theta_{P_1} = 42.27 + 180$$

$$2\theta_{P_1} = 222.27$$

$$\theta_{P_1} = \frac{222.27}{2}$$

$$\boxed{\theta_{P_1} = 111.135^\circ}$$ For Y-Axis

Now we have to find principal stresses as;

$$\sigma_1 = \frac{\sigma x + \sigma y}{2} + \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2}$$

$$\sigma_1 = -\frac{40+70}{2} + \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\boxed{\sigma_1 = 89.33 \text{ MPa}}$$

Also

$$\sigma_2 = \frac{\sigma x + \sigma y}{2} - \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau xy^2}$$

$$\sigma_2 = -\frac{40+70}{2} - \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\boxed{\sigma_2 = -59.33 \text{ MPa}}$$

We Must check which angle goes with which principal ~~axis~~ Stresses

$$\sigma x_1 = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2}\cos 2\theta + \tau xy \sin 2\theta$$

$$= -\frac{40+70}{2} + \left(\frac{-40-70}{2}\right)\cos 2(21.14) + (-50)\sin(21.14)$$

$$\boxed{\sigma x_1 = -59.33 \text{ MPa}}$$

Hence

$\delta_1 = 89.33\ MPa$ with $\theta_{P_1} = 111.135^0$

$\delta_2 = -59.33\ MPa$ with $\theta_{P_2} = 21.14^0$

Part: b:

Given data:

$\delta_x = -40\ MPa$

$\delta_y = 70\ MPa$

$\tau_{xy} = -50\ MPa$

Required data:

Max: In-Plane Shear Stress = ?

Max: Shear plane = ?

Solution:

First we find max: In plane Shear Stress

$$|\tau_{max\ in\ plane}| = \sqrt{\left(\frac{\delta_x - \delta_y}{2}\right)^2 + \tau_{xy}}$$

$$= \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$|\tau_{max\ in\ plane}| = 74.3\ MPa$$

$$\delta_{avg:} = \frac{\delta_x + \delta_y}{2} = \frac{-40+70}{2}$$

$$\boxed{\delta_{avg:} = 15}$$

Now we have to find Max: Shear plane

$$\tan 2\theta_s = -\frac{\left(\frac{\delta x - \delta y}{2}\right)}{\tau_{xy}}$$

$$\tan 2\theta_s = -\frac{\left(\frac{-40-70}{2}\right)}{-50}$$

$$\tan 2\theta_s = -1.1$$

$$\theta_s = 1/2 \tan^{-1}(1.1)$$

$$\boxed{\theta_s = -23.9^\circ}$$