

Iqra National University.
Linear Algebra Major Assignment for Spring 2020.
Semester - II, BS-SE.

Q1. Compute adjoint of;

(i) $A = \begin{bmatrix} 1 & 2 & \text{2nd-ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ \because 2nd-ID - 2nd number of your ID.

(ii). $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

Q2. Find the cofactors of A_{21} , A_{31} , A_{33} if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Q3. Find Eigen values and Eigen vectors if.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad \& \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Name : Junaid Ur Rehman

ID : 11484

Subject : Linear Algebra

Teacher : Sir Sakeel

NAME # Junaid-Ur-Rehman
ID # 11484
Linear Algebra

(1)

Q1# compute adjoint of ? 2nd ID = 1

(i) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

SOL# $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

So
 $A = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$$A_{11} = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = |6 - 1| = 5$$

$$A_{12} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \cancel{A_{12} [(3-4)]} [4 - 3] = 1$$

$$A_{13} = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} = A_{13} [2 - 3] = -1$$

$$A_{21} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = A_{21} [4 - 1] = 3$$

$$A_{22} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = A_{32} [2 - 3] = -1$$

$$A_{23} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = A_{23} [1 - 6] = -5$$

(2)

$$A_{31} = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} = A_{31} [2 - 3] = -1$$

$$A_{32} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = A_{31} [1 - 2] = -1$$

$$A_{33} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = A_{33} [3 - 4] = -1$$

$$\begin{bmatrix} 5 & -1 & -1 \\ -3 & -1 & +5 \\ -1 & +1 & -1 \end{bmatrix}$$

Cofactor matrix :-

$$A^T = \begin{bmatrix} 5 & -3 & -1 \\ -1 & -1 & 1 \\ -1 & +5 & -1 \end{bmatrix}$$

- adjoint matrix

(3)

Q1(ii)

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

Soln

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix} \begin{vmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{vmatrix}$$

So

$$B_{11} = \begin{bmatrix} -1 & 8 \\ -2 & 8 \end{bmatrix} = A_{11} [-16 - (-8)] = -8$$

$$A_3 B_{12} = \begin{bmatrix} 2 & 8 \\ 5 & 8 \end{bmatrix} = A_{12} [16 - 40] = -24$$

$$B_{13} = \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix} = [-4 - (-5)] = 1$$

$$B_{21} = \begin{bmatrix} 4 & 5 \\ -2 & 8 \end{bmatrix} = [32 - (-10)] = 42$$

$$A_{22} = \begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix} = [24 - 25] = -1$$

$$B_{23} = \begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} -6 & -20 \end{bmatrix} = -26$$

$$B_{31} = \begin{bmatrix} 4 & 5 \\ -1 & 8 \end{bmatrix} = \begin{bmatrix} 32 & -(-5) \end{bmatrix} = 37$$

$$B_{32} = \begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 24 & -10 \end{bmatrix} = 14$$

$$B_{33} = \begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & -8 \end{bmatrix} = -11$$

$$\begin{bmatrix} -8 & +24 & +1 \\ -42 & -1 & +26 \\ 37 & -14 & -11 \end{bmatrix} \text{ is factor of matrix}$$

$$B^T = \begin{bmatrix} -8 & -42 & 37 \\ +24 & -1 & -14 \\ +1 & +26 & -11 \end{bmatrix} \text{ is adjoint of matrix}$$

Q2 Find the cofactor of A_{11} , if (5)

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix}$$

$$= (-1)^3 (-4 - (-9))$$

$$= -1 \times (-4 + 9)$$

$$= -1 \times (5)$$

$$\boxed{A_{21} = -5}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= (-1)^4 (-2 - (9))$$

$$= (+1) (-2 - (9))$$

$$= (1) (-11)$$

$$\boxed{A_{31} = -11}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix}$$

$$= (-1)^6 (3 - (4))$$

$$= (1) \times (-1)$$

$$= (-1)$$

$$\boxed{A_{33} = -1}$$