Exam: FINAL

Subject: Probability & Statistics

Teacher Name: Sir. Dawood

Semester: 3rd

Name: AMIR ABBAS **ID: 15499**

Q.1

1. The sample space for experiment is

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$n(S) = 36$

Let $A = \{\text{The Sum is } 7\} \Rightarrow n(A) = 6$

$B = \{\text{The Sum is Even}\} \Rightarrow n(B) = 18$

$C = \{\text{The Sum is greater than } 8\} = 10$

$D = \{\text{Two dice have same outcomes}\} \Rightarrow n(D) = 6$

$P(A) = \frac{6}{36}$, $P(B) = \frac{18}{36}$, $P(C) = \frac{10}{36}$, $P(D) = \frac{6}{36}$

$P(A \cap B) = 0$, $P(A \cap C) = 0$, $A \cap D = \{\ \} \Rightarrow n(A \cap D) = 0$
$P(A \cap D) = 0$

1) $P(A/B) = \frac{P(A \cap B)}{P(B)} = 0$, ~~(2) P(A∩C) = 0~~

2) $P(A/C) = \frac{P(A \cap C)}{P(C)} = 0$

3) $P(A/D) = \frac{P(A \cap D)}{P(D)} = 0$

End Q.1

72 Show that in a single throw of two dice, the probability of throwing more than 7 is equal to that of throwing less than 7 and hence find the probability of throwing exactly 7 state clearly what assumption you are making solution

Answers :-

Sum of 2 has 1 way (1.1

Sum of 3 has 2 ways (1,2) and (2,1).

Sum of 4 has 3 ways 1,3,2 and 2,3,1 5 has 4 ways.

6 has 5 ways

8 has 5 ways (symmetry)

9 has 4 ways

10 has 3 ways

12 has 1 way

Those are 15/36 for each side with sum of 30/36

That leaves a 6/36 = 1/6 probability for a sum of 7.

(3)
Ans A&B play a game

(3)
Ans We observe that;

A:- There are two possible outcomes i.e. a will win or will not win the game.

B:- The probability of A's winning in each game is $p = 2/3$;

C):- The successive games are independently won or lost. &

D):- There are eigut games;

Therefore;
The binomial probability distribution with $n = 8$ & $p = 2/3$

is appropriate.

Let x denote the number of games won by A then

i) :- $P(X=4) = \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4 =$

$$\frac{1120}{6561} = 0.1707.$$

ii) $P(X \geq 4) = 1 - P(X<4)$;
($\because$ at least 4 means 4 or more).

$$= 1 - \sum_{x=0}^{3} \binom{8}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^6\right].$$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$\frac{577}{6561} = \frac{5984}{6561}$$

(4)

Ans :- Proof :-

Since the C_i,s from a partition of the sample space we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B) / C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)(B/C_i) P(C_i$$

:- (A and B are conditionally independent).

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i) P(B) P(C_i)$$

∵ (B is independent of all C_i,s $P(A \cap B) = P(B) \sum_{i=1}^{m} P(A/C_i) P(C$

$(A \cap B) = P(B)\,P(A)$

:- Law of total

lity

Hence A & B are

dent :

(5)

Ans Derivation of binomial probability Distribution :-

To derive a formula that gives the probability of success in n trails for a binomial experiment, we proceed as follows;

The experiment has n trails, each of which may result in S or F. The sample space has 2^n possible sample points or outcome, each outcome consisting of a sequence $(a_1, a_2 --- a_n)$, where each a_i, is either S or F. We desire to find the probability of these outcome according to the number of success.

First, we consider the probability of zero success, i.e, $P(X=0)$. In case of zero success, every trail result in F & the event consists of a sequence of n F's, i.e $\{FF \ldots F\}$. Because in each trial, $P(S) = p$ & $P(F) = q$ & trial are independent. so we apply the multiplicative law of probability for independent events and obtain.

$$P(FF \ldots F) = P(F)\,P(F) \ldots P(F) \quad (n \text{ times})$$
$$= q^n.$$

Since there is only one sequence of outcomes of n trials resulting in F's. therefore

$$P(X=0) = q^n.$$

Next, we consider the probability of one success, i.e.

$$P(X=1).$$

In this case, one trial results in S & the remaining (n-1) trials result in F's.

The event consisting of one S and (n-1) F's can occur in several d/f sequence. One such sequence is (SFF---F) & the probability for this sequence in pq^{n-1}.

Another possible sequence is {FFSF---F} & the probability for this sequence is {FFSF---F} & the probability for this sequence is the same as of the first sequence.

In other words, the probability for any possible sequence consisting of one S and (n-1) F's in pq^{n-1}.

But the number of mutually exclusive sequence in which one S & (n-1) F's can occur, is $\binom{n}{1}$. Therefore the probability of exactly one success for all possible sequence combined, is

$$P(X=1) = \binom{n}{1} pq^{n-1}.$$

The above argument may be repeated for $X = 2, 3, 4$ e.t.c.

Finally, we consider the general case; i.e. $X = x$. The probability of a sequence that has exactly x success and $(n-x)$ failures is $p^x q^{n-x}$ and there are $\binom{n}{x}$ d/f sequence in which x successes and $(n-x)$ failures can occur.

Therefore the probability of x successes in n trials is

$$P(X=x) = \binom{n}{x} p^x q^{n-x}, \text{ for } x = 1, 2, 3 \ldots, n.$$

Thus, we have obtained

The formula for the binomial probability distribution having n trials and probability p for success.

The binomial probability distribution derives its name from the fact that the probabilities $\binom{n}{x} \rightarrow$

$p^x q^{n-x}$, for $x = 0, 1, 2 \ldots n$ are the

successive terms of the binomial expansion of $(q+p)^n$, That is

1) Let X be a random variable with the binomial distribution $b(x; n, p)$ Then its mean & variable are given by $\mu = np$ & $\sigma^2 = npq$ respectively,

Now Mean, $\mu = E(X)$

$$\Rightarrow \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}, \text{ where } x = 0, 1, 2 \ldots n$$

$$\Rightarrow 0 \cdot q^n + 1 \cdot \binom{n}{1} q^{n-1} p + 2 \binom{n}{2} \rightarrow$$

$$q^{n-2} p^2 + \cdots + n p^2$$

$$\Rightarrow np \left[q^{n-1} + \binom{n-1}{1} q^{n-2} p + \binom{n-1}{2} \rightarrow \right.$$

$$\left. q^{n-3} p^2 + \cdots + p^{n-1} \right].$$

$$\Rightarrow np \, (q+p)^{n-1}.$$

$\Rightarrow np$, because $q + p = 1$.

Alternative Method :-

$$\text{Mean} = E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}.$$

But

$$x \binom{n}{x} = \frac{x(n)(n-1)!}{x(n-1)(n-x)!} = n \binom{n-1}{x-1}$$

$$E(X) = n \sum_{x=1}^{n} \binom{n-1}{x-1} p^x q^{n-x}, \text{ for } x = 1, 2 \text{ -- } n$$

(Since the first time in the summation being zero $(x=0)$ is omitted).

$$\Rightarrow np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} q^{(n-1)-(x-1)}$$

Substituting $y = x-1$ & $m = n-1$ in the summation, we get

$$E(X) = np \sum_{y=0}^{m} \binom{m}{y} p^y q^{m-y}$$

(as x ranges from 1 to n, so y $(=x-1)$ must range from 0 to $n-1$ i.e. m)

$= np$ ($\because$ Summation is the expansion of $(q+p)^m$).

Hence mean $= np$. In other words, the mean number of success is np.

Similarly the mean number of failures is nq.

By definition, the various σ^2, is given by

$$\sigma^2 = E[X+\mu]^2 = E(X^2) - [E(X)]^2$$

But $E(X)^2 = E[X(X-1)+X] = E[X(X-1)] + E(X)$.

$$= E[X(X-1)] + np.$$

Now $E[X(X-1)] = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x q^{n-x}$.

$$= \sum_{x=0}^{n} x(x-1) \frac{n(n-1)(n-2)}{x(x-1)(x-2)!\,(n-x)!}$$

$$\rightarrow p^2 p^{x-2} q^{n-2}.$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^{x-2} q^{n-x}$$

(x start at 2 since $x = 0, 1$ add nothing to the sum).

The term $(n-x)$ may be written as $\{(n-2)-(x-2)\}$.

Substituting $y = x-2$ & $m = n-2$ in the summation, we obtain

$$E[X(X-1)] = n(n-1)p^2 \sum_{y=0}^{m} \rightarrow$$

$$\rightarrow \frac{m!}{y!\,(m-y)!} p^y q^{m-y}.$$

$$\Rightarrow n(n-1)p^2 \sum_{y=0}^{m} \binom{m}{y} p^y q^{m-y}$$

$$\Rightarrow n(n-1)p^2$$ (summation is 1)

Thus $\sigma^2 = E(X)^2 - [E(X)]^2$

$$= E[X(X-1)] + E(X) - [E(X)]^2$$

$$= n(n-1)\,p^2 + np - (np)^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np - np^2 = np(1-p) = npq.$$

&

$$\sigma = \sqrt{npq}.$$

Hence the variation of the number of Success is npq & the standard deviation is $\sqrt{npq}$.

(6)

Ans The binomial dist is denoted & formulated by $f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}$

Where $x = 0, 1, 2 \ldots n$.

It shows only the probability of an individual.

While

Binomial frequency distribution:-

If the binomial probability distribution is multiplied by N, the number of experiment or set, the resulting distribution is known as the binomial frequency distribution.

Thus the expected frequency of x successes in N

$\binom{n}{x} p^x q^{n-x} \cdot 1$;

Should be noted that the ~~N~~ n independent trials constitute one experiment or one set.

Ans 07

Solution:-

Measure	Data set A	B	C	D
Coefficient of variation	$Cv = \frac{3}{45} \times 100$	$Cv = \frac{11}{60} \times 100$	$Cv = \frac{5}{50} \times 100$	$Cv = \frac{15}{25} \times$
	$Cv = 6.7$	$Cv = 18.3$	$Cv = 10$	$Cv = 60$