20/08/2020 ① Linear Algebra

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Q Ⓐ Let $A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{pmatrix}$ Identify the (3,2) entry of AB

Solution

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 4 \\ 3 & -1 \\ 2 & 2 \end{pmatrix}$$

Row 3 (A) Col 2 B

$$= \begin{bmatrix} 0 & 1, & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} = -5$$

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Labble the quadratic polynomial Interpolete the point (1, 3), (2, 4) (3, 4)

Solution.

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$
$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$
$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now

$$(x_1, y_1) = (1, 3) \quad (x_2, y_2) = (2, 4)$$
$$(x_3, y_3) = (3, 7)$$

put in

$$a_2 + a_1 + a_0 = 3$$
$$4a_2 + 2a_1 + a_0 = 4$$
$$9a_2 + 3a_1 + a_0 = 7$$

2

$$A_b = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array}\right)$$

$$\underset{\sim}{R} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & 8 \\ 0 & -6 & -8 & 20 \end{array}\right) \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$\underset{\sim}{R} = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right) \quad R_3 - 3R_2$$

So

$a_1 \quad a_2 + a_0 = 3$ —— 1

$-2a_1 - 3a_0 = -8$ —— 2

$a_0 = 4$ put in 2

$-2a_1 - \cancel{6a}12 - 8 = a_1 = 4/2 = -2$

put in 1

$a_2 - 2 + 4 = 5$

$a_2 = 1$

Solution

Since $|A^{-1}B^{T}| = |A^{-1}|\,|B^{t}|$

$$= \frac{1}{|A|}|B| \quad \text{bec } |B'| = |B|$$

So $= |A^{-1}B^{t}| = \frac{1}{|A|}|B|$

$$= \frac{1}{2}\cdot 3 = \frac{3}{2}$$

Q3(2) The Estimate the linear system of Eq

$$x + y + 2z = 1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

Solution

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right) \begin{array}{l} R - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/3 & 2 \\ 0 & -2 & -5 & 0 \end{array}\right] \cdot \frac{R_2}{-3}$$

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(4)

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 7/13 & 2 \\ 0 & 0 & -13/2 & 4 \end{array}\right] R_3 + 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & 1 & 8/13 \end{array}\right] R_3 \times \frac{1}{-13}$$

$x + y + 2z = 7$ — (i)

$y + 7/3 z = 2$ — (ii)

$z = -8/13$ — (iii)

Now put eq (iii) in eq (i)

$y + 7/3 \times \frac{8}{13} = 2$

$y - \frac{8}{39} = 2$

$y = 2 + \frac{8}{39}$

$y = \frac{78+8}{39} = \frac{86}{39}$

Now put value of y in (i)

$x + \frac{86}{39} + 2(-8/13) = 1$

$x + \frac{86}{39} - \frac{16}{13} = 1$

$x + 38/39 = 1$

$x = 1 - 38/39 = 1/39$ Ans

$A = \begin{pmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{pmatrix}$ Find A^{-1}

Solution

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{pmatrix}$$

$$3\begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$3(-4-6) + 2(-15-2) + 1(0-6)$$

$$|A| = -94$$

Now

$$A_{11} = (-1)^{1+2}\begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} = -8$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 2 \\ 6 & -3 \end{vmatrix} = -60$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 2 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 1 \\ 0 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 2 \\ 5 & 2 \end{vmatrix} = \cancel{28}\ -1$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{Adj } A = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{31} & A_{33} \end{pmatrix}^t$$

$$= \begin{pmatrix} A_{11} & A_{21} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{pmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{pmatrix}$$

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