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BSSE- 2nd Semester

Department of Computer Science
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Linear Algebra

Q#1

Q①. Ans:

$$\begin{bmatrix} 1 & 6 & 3 & 0 & 5 \\ 0 & 1 & 2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

$$R \begin{bmatrix} 1 & 0 & 0 & 0 & 37 \\ 0 & 1 & 2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \quad R_1 - 6R_2$$

$$R \begin{bmatrix} 1 & 0 & 0 & 0 & 37 \\ 0 & 1 & 0 & 0 & 18 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \quad R_2 - 2R_3$$

$$R \begin{bmatrix} 1 & 0 & 0 & 0 & 37 \\ 0 & 1 & 0 & 0 & 18 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \quad R_3 + R_2$$

$$R \begin{bmatrix} 1 & 0 & 0 & 0 & 37 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \quad R_2 - 3R_4$$

$$R \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \quad R_1 / \cancel{37} R_2$$

$$R \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \frac{R_4}{R_3}$$

Hence proved!

Q#2 Part (A)

Q No 2 (a)

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

(i) Transform first matrix into second

Sol

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$\underset{R}{\sim} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \quad R_3 - 2R_2$$

(ii) Transform 2nd matrix into 1st

$$= \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$2R_2 + R_3$

$$\underset{R}{\sim} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & 5 & -1 \end{bmatrix}$$

Q#2 Part (B)

QNo2 (b)

(a)

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & -n & 0 \\ 0 & 0 & 0 & e \end{bmatrix} \underset{R}{\sim} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & e \end{bmatrix} \begin{array}{l} \frac{R_1}{e} \\ \\ -R_3 \\ \\ \end{array}$$

It is the echelon form

(b)

$$\begin{bmatrix} 1 & 0 & n \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

It is ~~red~~ echelon form

(c)

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\underset{R}{\sim} \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\underset{R}{\sim} \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix} \; R_1/5$$

Q#2 Part (B)

$$C = R = \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} R2 - R3$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} R_3 - 4R_2$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \frac{R_1}{7/5}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} R_1 - R_2$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} R_2/R_1 \quad \text{Ans.}$$

(d)
$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_2 \leftrightarrow R_3 \quad \text{Ans.}$$

Q#1 (Part A): The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example.

Ans:

Row Echelon Form

A matrix is in row echelon form (ref) when it satisfies the following conditions.

- The first non-zero element in each row, called the leading entry, is 1.
- Each leading entry is in a column to the right of the leading entry in the previous row.
- Rows with all zero elements, if any, are below rows having a non-zero element.

Each of the matrices shown below are examples of matrices in row echelon form.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

A_{ref} B_{ref} C_{ref}

Reduced Row Echelon Form

A matrix is in **reduced row echelon form** when it satisfies the following conditions.

- The matrix satisfies conditions for a row echelon form.
- The leading entry in each row is the only non-zero entry in its column.

Each of the matrices shown below are examples of matrices in reduced row echelon form.

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

A_{rref} B_{rref} C_{rref}

Q#3 Part (B)

Q.No.3 (b)

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$$= \begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -6 & 0 & 0 \\ 1 & -4 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & 17 \\ 1 & 0 & 0 \\ 1 & -4 & 12 \end{bmatrix} \begin{matrix} 2R_1 - R_2 \\ \& \\ \frac{1}{-6} \times R_3 \end{matrix}$$

$$= \begin{bmatrix} 1 & 6 & 8 \\ 0 & 4 & 17 \\ 0 & 6 & 8 \\ 0 & 10 & -4 \end{bmatrix} \begin{matrix} R_1 - R_3 \\ \& \\ R_1 - R_4 \end{matrix}$$

$$= \begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & 17/4 \\ 0 & 3 & 4 \\ 0 & 5 & -2 \end{bmatrix} \begin{matrix} \frac{1}{2} \times R_3 \\ \frac{1}{2} \times R_4 \\ \frac{1}{4} \times R_2 \end{matrix}$$

$$= \begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & 17/4 \\ 0 & 0 & \frac{35}{4} \\ 0 & 0 & \frac{93}{4} \end{bmatrix} \begin{matrix} R_2 \times 3 - R_3 \\ 5R_2 - R_4 \end{matrix}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & 17/4 \\ 0 & 0 & 1 \\ 0 & 0 & 93/4 \end{bmatrix} \frac{4}{35} \times R_3$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & 17/4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{by } \frac{93}{4} \times R_3 - R_4$$

Answer