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Sec: B

Subject: Differential Equations.

Assignment:- 01

Submitted to,
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Application of ODE's and PDE's in Engineering.

ODE (ORDINARY DIFFERENTIAL EQUATION):-

An equation contains only ordinary derivates of one or more dependent variable of single independent variable.

For example:-

$dy/dx + 5y = e^x$, $(du/dt) = x + y$

PDE (PARTIAL DIFFERENTIAL EQUATION):-

An equation contains partial derivates of one or more dependent variable of two or more independent variables.

For example;

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 2\frac{\partial u}{\partial t} \qquad \frac{du}{dy} = -\frac{dv}{dx}$$

APPLICATION OF ODE;

- MODELLING WITH FIRST-ORDER EQUATION:-
  - Newton's Law of Cooling.
  - Electrical Circuits.
- MODELLING FREE MECHANICAL OSCILLATIONS.
  - No Damping.
  - Light Damping.
  - Heavy Damping.
- MODELLING FORCED MECHANICAL OSCILLATIONS
- COMPUTER EXERCISE OR ACTIVITY.

GAME DEVELOPMENT:-

↳ Game Theoretic models, building block Concept and many applications are Solve with differential Equation.

↳ Graphical interference of analyzing data and creating boomers based on Partial differential equation Solving with finite element method.

ROBOTIC INDUSTRIALIZATION.

- Auto motion and robotic technologies for customized component, module and building.
  prefabrication are based on differential equation.

MOTIVATING EXAMPLE:-

- Differential equations have wide applications in various engineering and Science disciplines. In general, modeling variations of physical quantity, such as temperature, pressure, displacement, velocity, stress, strain, or concentration of a pollutant, with the change of time t or differential equations. Similarly, studying the variation of a physical quantity on other physical quantities would lead to differential equations.

↳ For example the change of strain on stress for some viscoelastic materials follows a differential equation.

EXAMPLE OF PDE:-

PDES are used to model many systems in many different fields of Science and engineering.

Important Example:-

- Laplace Equation.
- Heat Equation.
- Wave Equation.

LAPLACE EQUATION:-

↳ Laplace Equation is used to describe the steady state distribution of heat in a body.

↳ Also used to describe the steady state distribution of electrical charge in a body.

$$\frac{\partial^2 v(x,y,z)}{\partial x^2} + \frac{\partial^2 v(x,y,z)}{\partial y^2} + \frac{\partial^2 v(x,y,z)}{\partial z^2} = 0$$

HEAT EQUATION:

↳ The function $u(x,y,z)$ is used to represent the temperature at time t in a physical body at a point with coordinates (x,y,z).

↳ α is the thermal diffusivity. it is sufficient to consider the case $\alpha = 1$.

$$\frac{\partial u(x,y,z,t)}{\partial t} = \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

WAVE EQUATION:-

↳ The function $u(x,y,z)$ it is used to represent the displacement at time t of a particle whose position at rest is (x,y,z)

↳ The constant c represents the propagation speed of the wave.

$$\frac{\partial^2 u(x,y,z,t)}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$