SESSIONAL ASSIGNMENT

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checking limit along $y=0$
So, we get

$$\lim_{x\to 0} \frac{\sin(x^2+0)}{x^2} = 0/0$$

by L. Hospital Rule.

$$\Rightarrow \lim_{x\to 0} \frac{\cos(x^2)\,2x}{2x} = 1 \quad — (3)$$

checking limit along $x=y$

$$\Rightarrow \lim_{y=x\to 0} \frac{\sin(x^2+x^2)}{x^2+x^2} = 0/0$$

By L.H.R

$$\Rightarrow \lim_{y=x\to 0} \frac{\cos(2x^2)\,4x}{4x} = 1 \quad — (4)$$

$\Rightarrow$

checking limit along $x=y^2$

$$\Rightarrow \lim_{(y^2,y^2)\to(0,0)} \frac{\sin((y^2)^2+y^2)}{(y^2)^2+y^2}$$

$$= \lim_{y\to 0} \frac{\sin(y^4+y^2)}{y^4+y^2} = 0/0$$

(8)

Question No. 5

Sol:-

$$4x^2 - y^2 + 3z^2 = 10$$

and $P(2, -3, 1)$

Now

let $F(x, y, z) = 4x^2 - y^2 + 3z^2 - 10$

$F_x = 8x \Rightarrow F_x(2, -3, 1) = 8(2) = \boxed{16}$

also, $F_y = -2y \Rightarrow F_y(2, -3, 1) = -2(-3) = \boxed{6}$

also $F_z = 6z \Rightarrow F_z(2, -3, 1) = 6(1) = \boxed{6}$

we know that

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 10$$

So, $16(x - 2) + 6(y + 3) + 6(z - 1) = 10$

$\Rightarrow 16x - 32 + 6y + 18 + 6z - 6 = 10$

$\Rightarrow 16x + 6y + 6z - 20 - 10 = 0$

$\Rightarrow 16x + 6y + 6z - 30 = 0$

$\Rightarrow 16x + 6y + 6z = 30$

$\Rightarrow 6z = -16x - 6y + 30$

$\Rightarrow \boxed{z = -\frac{8}{3}x - y + 5}$

which is the required ~~eq~~ eq of tangent plane.

Q 3:-

$$f(x,y) = e^x \sin y + e^y \cos x$$

Sol:

$$f_x = e^x \sin y - \sin x\, e^y$$

and $f_y = \cos y\, e^x + e^y \cos x$

Now 2nd derivative

$$\boxed{f_{xx} = e^x \sin y - \cos x\, e^y} \quad \text{——— (1)}$$

$$\boxed{f_{xy} = \cos y\, e^x - e^y \sin x} \quad \text{——— (2)}$$

$$f_{yy} = -\sin y\, e^x + e^y \cos x$$

$$\boxed{f_{yy} = e^y \cos x - \sin y\, e^x} \quad \text{——— (3)}$$

also,

$$f_{yx} = e^x \cos y - \sin x\, e^y \quad \text{—— (4)}$$

as $f_{xy} = f_{yx}$

so, 2nd order derivatives are

$$\boxed{\begin{aligned} f_{xx} &= e^x \sin y - \cos x\, e^y \\ f_{yy} &= e^y \cos x - \sin y\, e^x \\ f_{xy} &= f_{yx} = e^x \cos y - \sin x\, e^y \end{aligned}}$$

(1)

Question No. 1.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

Sol:-

checking limit along $x=0$

So,

$$\lim_{(0,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{\substack{y\to 0\\ x=0}} \frac{0\cdot y}{\sqrt{0^2+y^2}}$$

$$= 0 \quad \text{—— (1)}$$

checking limit along $y=0$

$$\lim_{(x,0)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = \lim_{\substack{x\to 0\\ y=0}} \frac{x\cdot 0}{\sqrt{x^2+0^2}}$$

$$= 0 \quad \text{—— (2)}$$

Now checking limit along $x=y$

$$\lim_{x=y\to 0} \frac{x\cdot x}{\sqrt{x^2+x^2}} = \frac{x^2}{\sqrt{2x^2}} = \frac{x^{\cancel{2}}}{\cancel{x}\sqrt{2}}$$

$$= 0 \quad \text{—— (3)}$$

Now checking limit along $x=y^2$

So,

$$\lim_{(x,y)\to(0,0)} \frac{y^2\cdot y}{\sqrt{(y^2)^2+y^2}} = \frac{y^3}{\sqrt{y^4+y^2}}$$

$$= \frac{y^3}{\sqrt{y^2(y^2+1)}}$$

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4. Question 4:-

$\vec{a} = i + 2j - k$ $\quad \vec{b} = -2i + 3k$

$c = 7j - 4k$

$$a \cdot (b \times c) = ?$$

Sol:-

$$a \cdot (b \times c) = \begin{vmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 0 & 7 & -4 \end{vmatrix}$$

Expand by Row 1

$$= 1 \begin{vmatrix} 0 & 3 \\ 7 & -4 \end{vmatrix} - 2 \begin{vmatrix} -2 & 3 \\ 0 & -4 \end{vmatrix} - 1 \begin{vmatrix} -2 & 0 \\ 0 & 7 \end{vmatrix}$$

$$= 1(0 - 21) - 2(8 - 0) - 1(-14 - 0)$$

$$= -21 - 16 + 14$$

$$= -23$$

(9)

Now the eq of Normal plane. As

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=d \quad \text{---} (*)$$

Now In the given eq

$$A=4, \quad B=-1, \quad C=3, \quad d=10$$

So, eq (*) becomes

$$4(x-2)+(-1)(y+3)+3(z-1)=10$$

$$\Rightarrow 4x-8-y-3+3z-3=10$$

$$\Rightarrow 4x-y+3z-8-6=10$$

$$\Rightarrow 4x-y+3z=10+8+6$$

$$\Rightarrow \boxed{4x-y+3z=24}$$

Question No. 2:

$$f(x,y) = \begin{cases} \frac{\sin(x^2+y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

Sol:-

we know that function is countinuous if $f(a,b) = \lim_{(x,y)\to(a,b)} f(x,y)$

Now

$f(0,0) = 1$ ——— (1)

also, we have to find

$\lim_{(x,y)\to(0,0)} f(x,y)$

checking limit along $x = 0$

$$\lim_{y\to 0} \frac{\sin(0^2+y^2)}{0+y^2} = \lim_{y\to 0} \frac{\sin y^2}{y^2} = \frac{0}{0}$$

Applying L. Hospital rule.

$$\Rightarrow \lim_{y\to 0} \frac{\cos y^2 \cdot \cancel{2y}}{\cancel{2y}} = 1 \quad \text{——— (2)}$$

$\Rightarrow$ ~~=a~~

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By L. H. R.

$$= \lim_{y \to 0} \frac{\cos(y^4 + y^2) \cdot (4y^3 + 2y)}{(4y^3 + 2y)} = 1 \quad \text{--- } (5)$$

By determining limit along different paths we conclude that

$$\lim_{(x,y) \to (0,0)} f(x,y) = 1 \quad \text{--- } (*)$$

~~thus~~ as

$$f(0,0) = \lim_{(x,y) \to (0,0)} f(x,y)$$

$$1 = 1$$

Hence the function is countinuous.

②

~~lim~~ $= \lim\limits_{(x,y)\to(0,0)} \dfrac{y^2 x}{y\sqrt{y^2+1}}$

$= \dfrac{0}{\sqrt{0+1}}$

$= 0$ —— (4)

Now checking limit along $y = mx$

So

$= \lim\limits_{(x,xm)\to(0,0)} \dfrac{mx\, x}{\sqrt{x^2+(mx)^2}}$

$= \lim\limits_{x\to 0} \dfrac{m x^2}{\sqrt{x^2+m^2x^2}}$

$= \lim\limits_{x\to 0} \dfrac{x^2 m}{x\sqrt{1+m^2}}$

Applying limit we get

$= 0$ —— (5)

thus we get

$$\boxed{\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0}$$