Date:___/___/___

Name	Hidayat Khan
ID	17024
program	BSSE
Semester	3rd
paper	probability & Statistics
Teacher	Sir Daud.

Q1 A man throws two fair dice what is the conditional probability that the sum of the two dice will be 7 given that:

1 The Sum is even

2 The sum is greater than 8

3 The two dice had the same outcomes.

Solution

The simple space for this experiment is

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2),\ (2,3)\ (2,4)\ (2,5)\ (2,6)$

$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)\ (4,1)$
$(4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)\ (5,1)$
$(5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)\ (6,1)$
$(6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

let $A = \{$ The Sum is $7 \}$

B= {The sum is even}

C= {The sum is greater than 8}

and d= {The two dice lead the same outcome}

then

A= {(1,6) (2,5) (3,4) (4,3) (5,2) (6,1) (3,3)}

B= {(1,3) (1,5) (2,2) (2,4) (2,6) (3,1) (3,5) (4,2) ..... (6,6)}

C= {(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)}

$(A \cap B) = \phi$

$(A \cap C) = \phi$

$(A \cap D) = \phi$

$P(A) = 6/8$ $(AB) = 18/36$, $P(C) = 10/36$

$P(D) = 6/36$

$P(A \cap B) = 6/36$, $P(A \cap C) = 6/36$ and $P(A \cap D) = 0$

Hence

$$P(A/B) = P(A \cap B) = 6/36 \times \frac{36}{18} = 1/3$$

$$P(A/C) = P\frac{(A \cap C)}{P(C)} = 6/36 \times 36/10 = 3/5$$

$$P(A/D) = P\frac{(A \cap D)}{P(D)} = 0 \times \frac{36}{6} = 0$$

Q2 Show that in a single throw of two dice, the probability of throwing more than 7 is equal to that of throwing less than 7 and hence find the probability of throwing exactly 7 state clearly what assumption you are making solution

Answers :-

Sum of 2 has 1 way (1.1
Sum of 3 has 2 ways (1,2) and (2,1).
Sum of 4 has 3 ways 1,3,2 and 2,3,1 5 has 4 ways.
6 has 5 ways
8 has 5 ways (symmetry)
9 has 4 ways
10 has 3 ways
12 has 1 way

Those are 15/36 for each side with sum of 30/36

That leaves a 6/36 = 1/6 probability for a sum of 7.

Q3 A and B play a game in which A's probability of wining is 2/3 In a series of 8 games. what is the probability that A will win

1) Exactly four (4) games

2) At least 4 games

3) from 3 to 6 games

Soull Solution

we are given that

$p = 2/3 \quad n = 8$

$q = 1 - p$

$q = 1 - 2/3 \quad q = 1/3$

let "x" be the numbers of games won by then

Exactly 4 games $\rightarrow p(x=4)$

$= (8/4)(2/3)^4(1/3)^4$

$\frac{1120}{6561} = \boxed{0.1707}$

(2) At least 4 games $P(\geq 4)$

$= 1 - P(x < 4)$

$1 - \sum^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$

$1 - \left[\left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) + 28 \left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5}\right],$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561} \Rightarrow \frac{5984}{6561}$$

$$= 0.9121$$

$$P(x \geq 6)$$

$$\sum_{x=6}^{8} \binom{8}{x} (2/3)^{x} (1/3)^{8-x}$$

$$= \binom{8}{6}\left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)^{2} + \left(8/7\right)\left(2/3\right)^{7} \left(\tfrac{1}{3}\right) + \left(8/8\right)\left(2/3\right)^{8} \left(\tfrac{1}{3}\right)^{0}$$

$$= \frac{64}{6561}\left(28 + 16 + 4\right)$$

$$\frac{64 \times 48}{6561} = \frac{1024}{6561} = 0.4682$$

iv $p(3 < x \leq 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{3}{2}\right)^{4}\left(\frac{1}{3}\right)^{4}$$

$$+ \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3}$$

$$+ \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

Q4 proof

Since the C_i,s from = partition of the sample space we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B) C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1} P(A \cap B) C_i) P(C_i)$$

(A and B are ~~...~~ conditionally ~~...~~ independent)

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i) P(B) P(C_i)$$

$$P(A \cap B) = P(B) \sum P(A/C_i) P(C_i)$$

$$P(A \cap B) = P(B) (P(A)$$

law of that probability

Hence A and B are independent

Q5 Binomial distribution

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

For $x = 0, 1, 2, \ldots\ldots n$

$\mu = np$ // mean

$\sigma^2 = np(1-p)$ // variance

A binomial random variable can be thought of as 'the sum of n independant Bernoulli random variables each with mean p and variable p (1-p)

Let $V_1, \ldots\ldots V_n$ be independant Bernoulli random variables

$E(V_i) = p$ & $Var(V_i) = p(1-p)$

$X = V_1 + \cdots\cdots + V_n$
$var(X)$ $var(V_2) + \cdots\cdots$
$var(V_n)$

The Binomial Theorm

$$(a+b)^m = \sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y}$$

$$E(x) = \sum x\, p(x)$$

$$= \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,((n-1)-(x-1))!} p^{x-1} (1-p)^{x}$$

$m = (n-1)$, $y = (x-1)$

$$= np \sum_{y=0}^{m} \frac{m}{y!\,(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \binom{m}{y} (p^y (1-p))^{m-y}$$

now

$$Var(x)\ E[(x-\mu)^2]$$

$$= \sum (x-\mu)^2 \{p(x)$$

$$E[(x-\mu)^2] = E(x^2) - [E(x)]^2$$

$$E[x(x-1)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^{x}(1-p)^{n-x}$$

$$E[X(x-1)] = n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2}(1-p)$$

By Binomial theom

$$E(x(x-1)] = n(n-1)p^2$$

$$E(x^2 - x) = n(n-1)p^2$$

$$E(x^2) - E(x) = n(n-1)p^2$$

Since $E(x) = np$, which is mean of binomial

$$E(x^2) = n(n-1)p^2$$

$$E(x^2) = n(n-1)p^2 + np$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$var(x) = np[(n-1)p + 1 - np]$$

This is variance of binomial distribution

Now $var(x)\ E[(x-\mu)^2]$

$$= \Sigma (x-\mu)^2 p(x)$$

$$E[(x-\mu)^2 = E(x^2) - [E(x)]^2$$

$$E[x(x-2)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^n$$

$$E[x(x-1)] = n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{(n-2)-(x-2)}$$

By binomial theorem

$E(x(x-1)] = n(n-1)p^2$

$E(x^2 - x) = n(n-1)p^2$

$E(x^2) - E(x) = n(n-1)p^2$

$E(x^2) - E(x) = n(n-1)p^2$

Since $E(x) = np$ which is mean of binomial

$E(x^2) = n(n-1)p^2$

Q6

The Binomial list is denoted and formulated by $f(x) = p(X = x) = \binom{n}{x} p^x q^{n-x}$

where $x = 0, 1, 2, \ldots\ldots n$

it shows only the probability of an individual

While

Binomial frequency Distribution

if the binomial probability is multiplied by N the number of experiments or sets, the resulting distribution is known as the binomial frequency distribution. Thus the expected frequency of x success is N experiments is $N \binom{n}{x} p^x q^{n-x}$ should be noted that the n independent trials constitute one experiment or one set

Q7 Solution

Measure	Data set A	B	C	D
Coefficient of verification	$CV = \frac{3}{45} \times 100$	$CV = \frac{11}{60} \times 100$	$CV \frac{5}{30} \times 100$	$CV = \frac{18}{22} \times 100$
	$CV = 6.7$	$CV = 8.3$	$CV = 10$	$CV = 60$