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Subject # Applied Calculus

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MID TERM Exam

Q1) The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

a) State any point of discontinuity.

b) Find, if they exist

1. $\lim\limits_{t \to 3} g$

Solution:-

To check possibility of the discontinuity of the function is at $t = 0$ & 4.

→ First at $t = 0$

$$g(t) = t^2$$
$$g(0) = 0^2 = 0$$

For R.H.L :-

$$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$$
$$= \lim_{h \to 0} 1 + h^2 + 2h$$

Apply limits:

$$1 + 0^2 + 2(0)$$
$$= 1$$

For L.H.L.

$$\lim_{h \to 0} g(1-h) = 2t+3$$
$$= \lim_{h \to 0} 2(1-h) + 3$$

$= \lim_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

$R.H.L \neq L.H.L = g(t) = 5$

Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

For R.H.L.

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$

$= \lim_{h \to 0} 2 + 2h + 3$

Apply limits

$= 2 + 2(0) + 3 \Rightarrow 5$

For L.H.L

$\lim_{h \to 0} g(1-h) = 12$

$g(4) = R.H.L \neq L.H.L$

Point of discontinuity is at $t = 4$

Q1 Part B.

Find, if they exist.

(i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L

$\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limits

$= 1 + 3^2 + 2(3) \Rightarrow 16$

L.H.L :-

$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$

$= \lim_{h \to 3} 2(1-h) + 3$

$= \lim_{h \to 3} 2 - 2h + 3$

Apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L

(do not exist since L.H.L is -ve)

Q2 Find the Mauclarins Series for

$y(x) = x^2 + \sin x$

Solution:-

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

let

	Put $x = 0$ in all function
$y(x) = x^2 + \sin x$	$y(0) = 0$
$y'(x) = 2x + \cos x$	$y'(0) = 1$
$y''(x) = 2 - \sin x$	$y''(0) = 2$
$y'''(x) = -\cos x$	$y'''(0) = -1$

Putting value in formula

$$= 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1)$$

$$= 0 + x + x^2 - \frac{x^3}{6}$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} \text{ Ans.}$$

Q3 Find y'' given

$$1 + xy = x^2 + y^2$$

Solution:-

$$= \quad 1 + xy = x^2 + y^2 \longrightarrow ①$$

diff eq ① b/s w.r.t "x"

$$= d/dx\,(1 + xy) = d/dx\,(x^2 + y^2)$$

$$= d/dx\,(1) + d/dx\,(xy) = d/dx\,x^2 + d/dx\,y^2$$

$$= 0 + (x \cdot d/dx\,y + y \cdot d/dx\,x) = 2x + 2y \cdot dy/dx$$

$$= x \cdot d/dx + y\,(1) = 2 + 2y \cdot dy/dx$$

$$= x \cdot y' + y = 2x + 2y \cdot y'$$

$$= x \cdot y' + y = 2x + 2y \cdot y$$

$$= x \cdot y' - 2y \cdot y' = 2x - y$$

$$= y'\,(x - 2y) = (2x - y)$$

$$= y' = \frac{2x - y}{x - 2y}$$

Differ again w.r.t "x"

$$y'' = d/dx\left(\frac{2x - y}{x - 2y}\right)$$

$$= \frac{(x - 2y)\,d/dx\,(2x - y) - (2x - y)\,d/dx\,(x - 2y)}{(x - 2y)^2}$$

$$= \frac{(x-2y)(2)(-dy/dx) - (2x-y)(1-2\,dy/dx)}{(x-2y)^2}$$

$$= \frac{(2x-4y)(-y') - (2x - 4x\,dy/dx - 4 + 2y\,y')}{(x-2y)^2}$$

$$= y''(x-2y)^2 = 2xy' + 2yy' - 2x + y$$

$$= y''(x-2y)^2 = \left(\frac{2x-y}{x-2y}\right)(2x-2y) - 2x + y$$

$$y'' = \frac{\left(\frac{2x-y}{x-2y}\right)(2x+2y) - 2x + y}{(x-2y)^2} \quad \text{Ans}$$

(ii) Find y' by using logarithmic differentiation.

$$y = x^3(1+x)^9 e^{6x}$$

Solution:- Taking ln on b/s

$$\ln y = \ln(x^3(1+x)^9 \cdot e^{6x}$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x \ln e$$

Diff w.r.t x

$$d/dx \ln y = d/dx\, 3\ln x + d/dx\, 9\ln(1+x) + d/dx\, ^{6x}\ln e$$

$$1/y\, dy/dx = 3/x + 9/1+x + 1/e^{6x} \cdot 6$$

$$dy/dx = y\,(3/x + 9/1+x + 6/e^{6x})$$

$$dy/dx = x^3(1+x)^9 e^{6x}(3/x + 9/1+x + 6/e^{6x}) \text{ Ans.}$$