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Subject ; Differential Eq:

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Q1 ⓐ $w = \sin(x+ct) + \cos(2x+2ct)$.

Given

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} \longrightarrow ①$$

Now

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct) + \cos(2x+2ct)\right)$$

$$= \frac{\partial}{\partial t}\left(\sin(x+ct)\right) + \frac{\partial}{\partial t}\left(\cos(2x+2ct)\right)$$

$$\frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$$

Now

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left[c\cos(x+ct) - 2c\sin(2x+2ct)\right)$$

$$\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)$$

Now

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct) + \cos(2x+2ct)\right)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right).$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

(1) $\Rightarrow$

$$-c^2\sin(x+ct)-4c^2\cos(2x+2ct)=c^2\left[-\sin(x+ct)-4\cos(2x+2ct)\right]$$

$$-c^2\,\cancel{\sin}(x+ct)-4c^2\cos(\cancel{2x}+2ct)=-c\,\cancel{\sin}(x+ct)-4c^2\,\cancel{\cos}(x+2ct)$$

$$0=0\quad\boxed{\text{Satisfied}}$$

(ii) $w=\tan(2x+ct)$

Now $\frac{\partial w}{\partial t}=c\sec^2(2x+ct)$

& $\frac{\partial^2 w}{\partial t^2}=\frac{\partial}{\partial t}\left(c\sec^2(2x+ct)\right)$

$=c^2 2\cdot 2\sec^2(2x+ct)\tan(2x+ct)$

Now $\frac{\partial w}{\partial x}=2\sec^2(2x+ct)$

$\frac{\partial^2 w}{\partial x^2}=4\sec^2(2x+ct)\tan(2x+ct)$

(1) $\Rightarrow$ $4c^2\,\cancel{\sec^2}(2x+ct)\tan(2x+ct)=4c^2\,\cancel{\sec^2}(2x+ct)\tan(2x+ct)$

$$0=0\quad\boxed{\text{Satisfied}}$$

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Q:2 Given function is

$$F(x) = \begin{cases} x ; & -\pi < x \leq 0 \\ 2x ; & 0 \leq x \leq \pi . \end{cases}$$

We have to find the Fourier co-efficient, a_0, a_n & b_n

Now

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\,dx + \frac{1}{\pi} \int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi} \left[\frac{x^2}{2}\right]^{\pi}$$

$$= \frac{1}{\pi} \left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi} \left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = -\frac{\pi}{2} + \pi = \pi/2} \rightarrow (1)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx\,dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} (x \cos nx\,dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos nx)\,dx$$

$$= \frac{1}{\pi} \left[x\left(\frac{\sin x}{n}\right) - \left(-\frac{\cos x}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right)-\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2}-\frac{\cos n\pi}{n^2}\right]+\frac{2}{\pi}\left[\frac{\cos n\pi}{n^2}-\frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(1)^n+2(-1)^n-2}{n^2}\right] = \frac{(-1)^n-1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases}\frac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even}\end{cases}} \rightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right)-\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$(3) \leftarrow b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right]+\frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] =$$

$$-3\cos n\pi = \frac{3(-1)^{n+1}}{n}$$

So the required fourier series is

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)^2} + 3 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

~~Again use the another initial condition~~

Q:-3

<u>Given</u> $y'' - 4y' + 13y = 8 \sin 3x$.

We have to find $y = y_c + y_p$

For y_c The characteristic (auxilary Eqn) Eqn is:

$$m^2 - 4m + 13 = 0$$

$$\Rightarrow m = \frac{4 \pm \sqrt{16 - 52}}{2} \Rightarrow m = \frac{4+6i}{2}$$

$$m = 2 \pm 3i \qquad \alpha = 2 \text{ \& } \beta = 3$$

So $y_c = e^{2x} \{ c_1 \cos 3x + c_2 \sin 3x \}$

For y_p Let

$$y_p = I_{mag} \cdot \left(\frac{1}{m^2 - 4m + 13} 8 e^{3ix} \right)$$

For y_p Let

$$y_p = 8\,\text{Imag}\ \frac{e^{3ix}}{-9-12i+13}$$

$$= 8\,\text{Imag}\ \frac{e^{3ix}}{4-12i}$$

$$y_p = 2\,\text{Imag}\ \frac{e^{3ix}}{(1-3i)} \times \frac{(1+3i)}{(1+3i)}$$

$$y_p = 2\,\text{Imag}\ \frac{(1+3i)(e^{3ix})}{(1)^2-(3i)^2}$$

$$y_p = 2\,\text{Imag}\ \frac{(1+3i)(e^{3ix})}{10}$$

$$y_p = \frac{2}{10}\left(\text{Imag}\,(1+3i)\,(\cos 3x + i\sin 3x)\right)$$

$$y_p = \frac{2}{10}\left(\sin 3x + 3\cos 3x\right)$$

So the general solution is

$$y = y_c + y_p$$

$$y = C_1 e^{2x}\cos 3x + C_2 e^{2x}\sin 3x + \frac{2}{10}\left(\sin 3x + 3\cos 3x\right)$$

Now use initial condition : $y(0) = 2$

$$y(0) = C_1 e^{(0)} \cos(0) + C_2 e^{(0)} \sin(0) + \frac{2}{10}\left(\sin(0) + 3\cos(0)\right)$$

$$1 = C_1(1) + 0 + 0 + \frac{2}{10}\left(3(1)\right)$$

$$1 = C_1 + \frac{6}{10} \Rightarrow \boxed{C_1 = 1 - \frac{6}{10} = \frac{4}{10} = \frac{2}{5}}$$

Again use the Another Initial condition

$$y'(0) = 2$$

So
$$y' = C_1 2 e^{2x} \cos 3x + C_1 e^{2x}(-3\sin 3x)$$
$$+ C_2 2 e^{2x} \sin 3x + C_2 e^{2x}(3\cos 3x) + \frac{2}{10}\left(\cos 3x - 3\sin 3x\right)$$

$$y'(0) = C_1 2 e^{(0)} \cos(0) + C_1 e^{(0)}\left(-3\sin(0)\right)$$
$$+ C_2 2 e^{(0)} \sin(0) + C_2 e^{(0)}\left(3\cos(0)\right) + \frac{2}{10}\left(\cos(0) - 3\sin(0)\right)$$

$$2 = 2C_1 + 0 + 0 + C_2 3(1) + \frac{2}{10}\left(1 - 3(0)\right)$$

$$2 = 2C_1 + 3C_2 + \frac{2}{10}$$

$$2 = 2\left(\frac{2}{5}\right) + 3C_2 + \frac{2}{10} \qquad \boxed{\text{use } C_1 = \frac{2}{5}}$$

$$\frac{1}{3}\left(2 - \frac{4}{5} - \frac{2}{10}\right) = C_2 \Rightarrow C_2 = \frac{1}{3}\left(\frac{20-8-2}{10}\right) = \frac{1}{3}$$

So The General Solution is

$$y = \frac{2}{5} e^{2x} \cos 3x + \frac{1}{3} e^{2x}\left[\sin 3x + \frac{2}{10}\right]\left[\sin 3x + 3\cos 3x\right]$$

by The Required Solution.

$\sim\!\sim\; x \;\sim\!\sim\; x \;\sim$

Q: 4

$$\left(D^2 - DD'\right) z = \cos x \cos 2y$$

The given PDE can be rewrite as:

$$D\left(D - D'\right) u = \cos x \cos 2y$$

its CF is given by

$$CF = \phi_1(y) + \phi_2(y + x)$$

While its PI is given by:

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)} \cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y).$$

Hence the complete solution of the given PDE is given by.

$$u = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

Ans