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SECTION : A

PAPER : DIFFRENTIAL EQUATION

QUESTION #1

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2)$$

Solution:

$y(0) = 0$ so $x=0$, $y=0$

$$dy = e^y e^{-t} \sec(y)(1+t^2)\,dt$$

$$\frac{1}{e^y \cdot \sec(y)}\,dy = (1+t^2)e^{-t}dt$$

As; $\cos(y) = \dfrac{1}{\sec(y)}$

$$\int_* e^{-y} \cos y\,dy = \int (1+t^2)\,e^{-t}dt$$

using integration by parts.

$$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \frac{d}{dy} e^{-y}\right) =$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \frac{d}{dt}(1+t^2) \rightarrow 1\right.$$

L.H.S

$$e^{-y}\int \cos y\,du - \int\left(\int \cos y \cdot \frac{d}{dy} e^{-y}\right)$$

$$e^{-y}\sin y - \int(\sin y \cdot e^{-y} \cdot (-1))$$

$$e^{-y}\sin y + \int(\sin y \cdot e^{-y})$$

$$e^{-y}\sin y + \int(e^{-y}\sin y)$$

Again using integration by parts.

$$e^{-y}\sin y + e^{-y}(-\cos y) - \int\left(\int \sin y \frac{d}{dy} e^{-y}\right)$$

$$e^{-y}\sin y + e^{-y}(-\cos y) - \int\left(\int \cancel{\sin y}\ \frac{e^{-y}}{-1}\right)$$

$$e^{-y}\sin y - e^{-y}\cos y - \int(\cos y\ e^{-y})$$

Since $\int(\cos y\, e^{-y}) = \text{LHS}$.

since it is again same to the first one so L.H.S will become

$$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S.$$

$$2LHS = e^{-y}(\sin y - \cos y)$$

$$LHS = \frac{e^{-y}(\sin y - \cos y)}{2}$$

Now RHS.

$$= \int (1+t^2)\, e^{-t}\, dt$$

$$= (1+t^2)\int e^{-t} - \int\left(\int e^{-t} \frac{d}{dt}(1+t^2)\right)$$

$$= -(1+t^2) + \int((2t)\, e^{-t})$$

again using integration by part.

$$= -(1+t^2)\, e^{-t} + \left(2t\int e^{-t} - \int\left(\int e^{-t} \frac{d}{dt} 2t\right)\right)$$

$$= -(1+t^2)\, e^{-t} + \left(-2te^{-t} - \int(e^{-t}\, 2)\right)$$

$$= -(1+t^2)\, e^{-t}\left(-2te^{-t} - 2e^{-t}\right) + C$$

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$= -(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + c$

$= -e^{-t} - e^{-t}t^2 - 2te^{-t} - 2e^{-t} + C$

$= -(t^2+2t+3)e^{-t} + C = R.H.S$

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + c$$

we know that

$t = 0 \qquad y = 0$

put it above

$= \frac{1}{2}(0-1) = -3 + C$

$C = 5/2$

put value of c.

$$\boxed{\frac{e^{-y}(\sin y - \cos y)}{2} = -(x^2+2t+3)e^{-t} + 5/2}$$

QUESTION #2:

$$\left(\sqrt{x+y} + \sqrt{x-y}\right)dx - \left(\sqrt{x+y} - \sqrt{x-y}\right) dy = 0,$$

SOLUTION:

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \rightarrow (1)$$

This is homogenius Diffrential or in x and y to solve this put:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

This eq (1) becomes,

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+v + 1-v + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}(1 + \sqrt{1-v^2})}{\cancel{2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} \cancel{+v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking integral on b/s.

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

put $\quad 1+\sqrt{1-v^2} = t$

$$= \tfrac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c.$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\cancel{\ln}(1+\sqrt{1-v^2}) = \cancel{\ln}(cx)^{-1}$$

$$1+\sqrt{1-v^2} = 1/cx$$

$$1+\sqrt{\frac{x^2-v^2}{x^2}} = 1/cx$$

$$x + \sqrt{x^2 - y^2} = \frac{1}{c}$$

$$x + \sqrt{x^2 - y^2} = c_1 \quad \because \ 1/c = c_1$$

which is required solution.

QUESTION #3

$(D^4+D^2)y = 3x^2 + 4\sin x - 2\cos x.$

Solution:

$(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$

$F(D)y = f(x)$

it is non homogenous linear equation.

$y = y_c + y_p$ —— (i)

complementry Solution. y_c

$D^4 - D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow D = 0$

$D^2+1 = 0 \Rightarrow D^2 = -1$

$D = \sqrt{-1} \Rightarrow \boxed{D = i}$ or $\boxed{D = 0}$

roots are Real and complex

$y_c = c_1 e^{0x} + e^{0x}(C_2 \cos x + C_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x$

$y_p = \frac{1}{f(D)} F(x)$

$y_p = \frac{1}{D^4 + D^2} (3x^2 - 4\sin x - 2\cos x)$

$= \frac{3x}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(D) = 0$

So

$f(D) = 4D^3 - 2D$

Now also for $D = 0 \Rightarrow f(D) = 0$

again differentially

$f''(D) = 12D + 2$

So for $D = 0$

$f''(0) = 12(0) + 2 = 2.$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} . 4\sin x - \frac{x^2}{12D+2}$

$. 2\cos x.$

So putting $D=0$ in all.

$$y_p = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}.$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2\cos x}{2}.$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

putting in equation (1)

$$y = C_1 + C_2\cos x + C_3\cos x + 3/2\, x^4 + 2x^2\sin x$$

$$- x^2\cos x$$

$$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + 3/2\, x^4$$