

Syed Jawwad

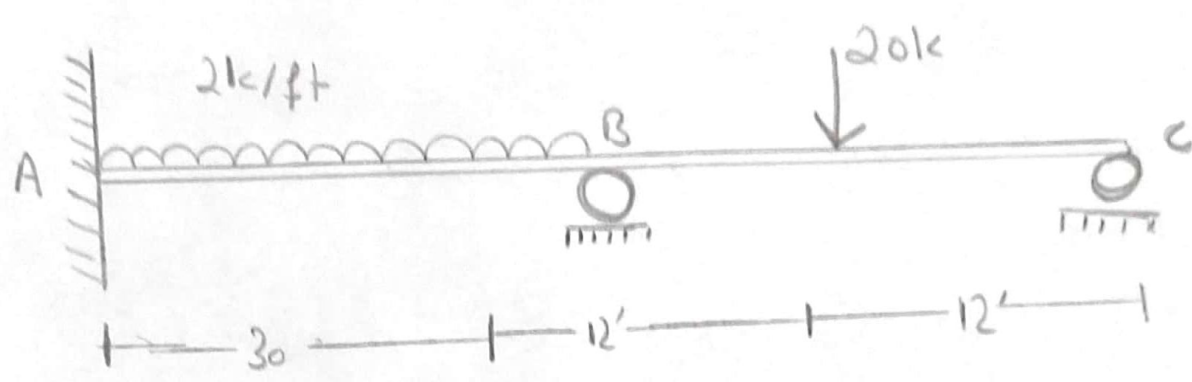
7386

STRUCTURE ANALYSIS - 2

MID EXAM

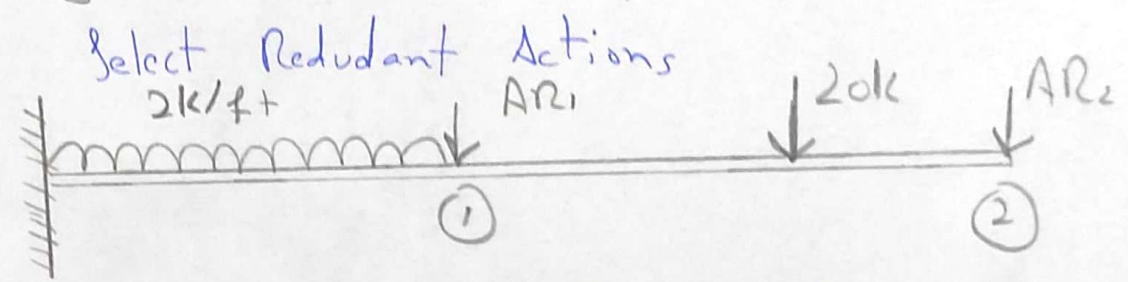
SUMMER SEMESTER

Q # 7



Sol:-
Structural Indeterminacy = 2°

Step # 1:-

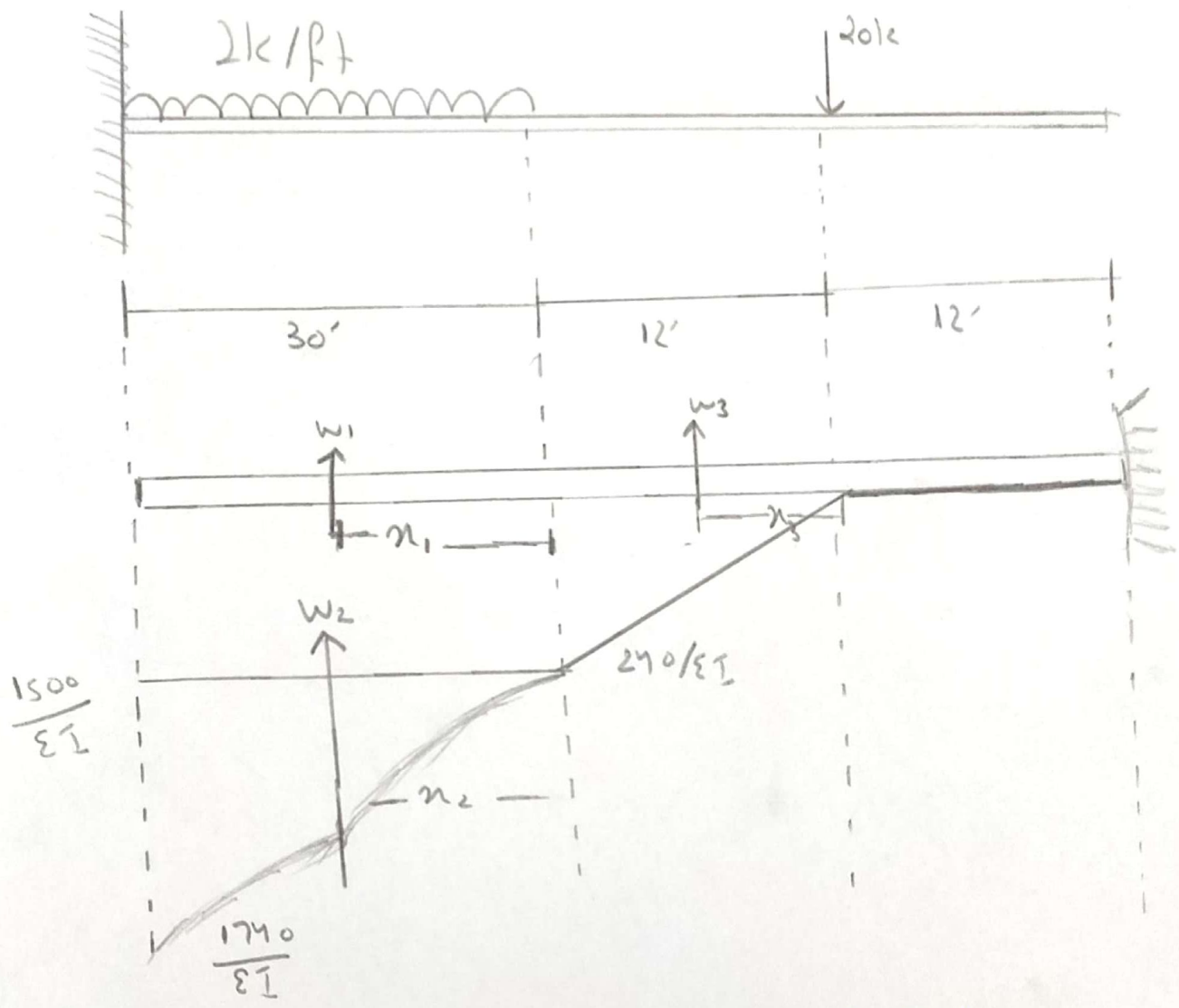


$$\begin{bmatrix} DR_{s1} \\ DR_{s2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = [DRL + F] \times [AR]$$

STEP # 2

Compute the values of [DRL]



$$w_1 = 1500 \times 30 = 45000$$

$$20 \times 12 = 240$$

$$20 \times (12 + 30) \neq$$

$$2 \times 30 \times 15 = 1740$$

$$w_2 = \frac{1}{3} \times 50 \times 240 = 2400$$

$$w_3 = \frac{1}{2} \times 12 \times 240 = 1440$$

$$x_1 = b/2 = 30/2 = 15'$$

$$x_2 = \frac{3}{n+2} \times L = \frac{3}{2+2} \times 30 = 22.5'$$

$$x_3 = \frac{2}{3} \times L = \frac{2}{3} \times 30 = 20'$$

Now finding DRL:-

$$DRL_2 = w_1 \times (n_1 + 24) + w_2 \times (n_2 + 24) + w_3 \times (n_3 + 12)$$

$$= 45000 (15 + 24) + 2400 (22.5 + 24) + 1440 (8 + 12)$$

$$= 1755000 + 111600 + 28800$$

$$DRL_2 = \frac{1895400}{\Sigma I}$$

$$DRL_1 = w_1 (n_1) + w_2 (n_2)$$

$$= 45000 (15) + 2400 (22.5)$$

$$= 675000 + 54000$$

$$= 729000$$

So,

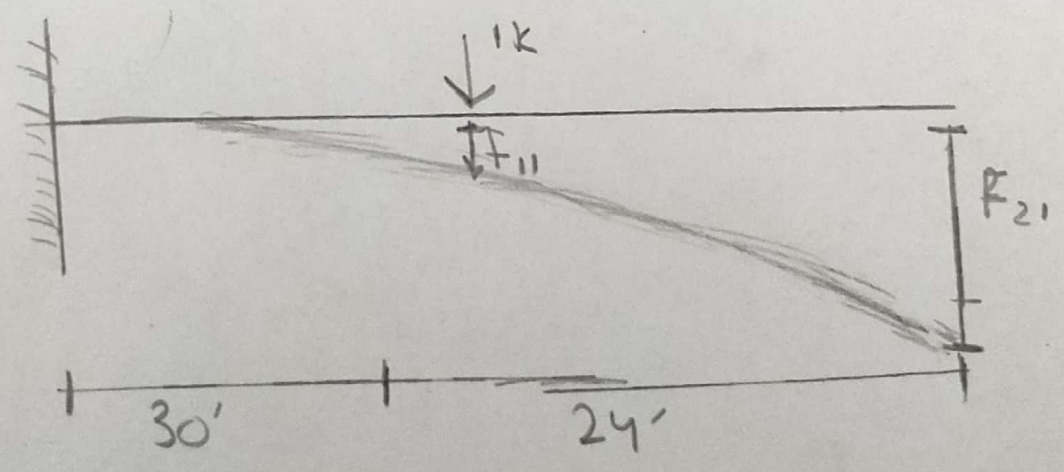
$$DRL = \frac{1}{\Sigma I} \begin{bmatrix} 729000 \\ 1895400 \end{bmatrix}$$

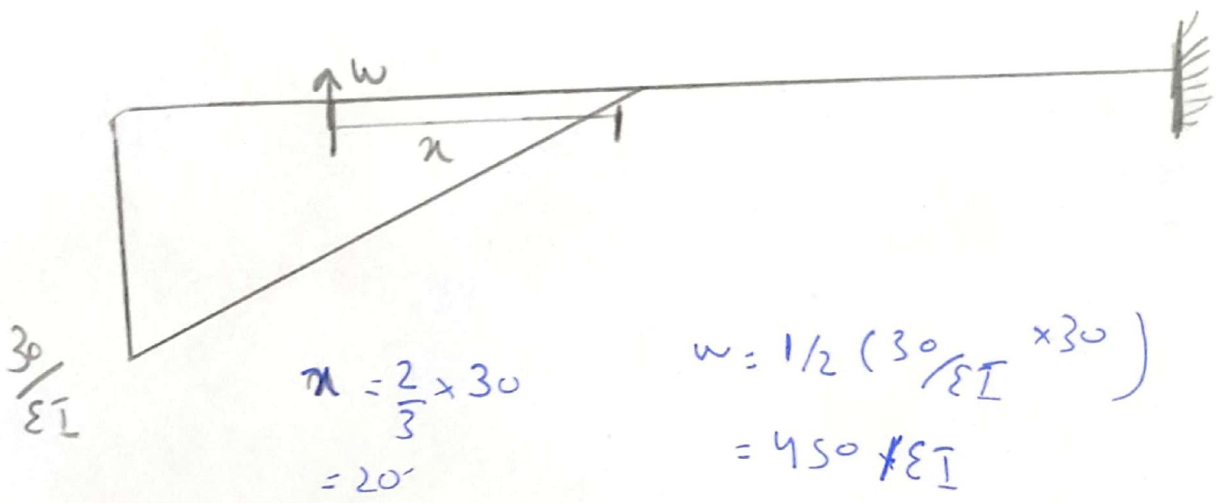
Step #3

FLEXIBILITY MATRIX

$$[F]_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

a) Applying Unit loaded on AR1



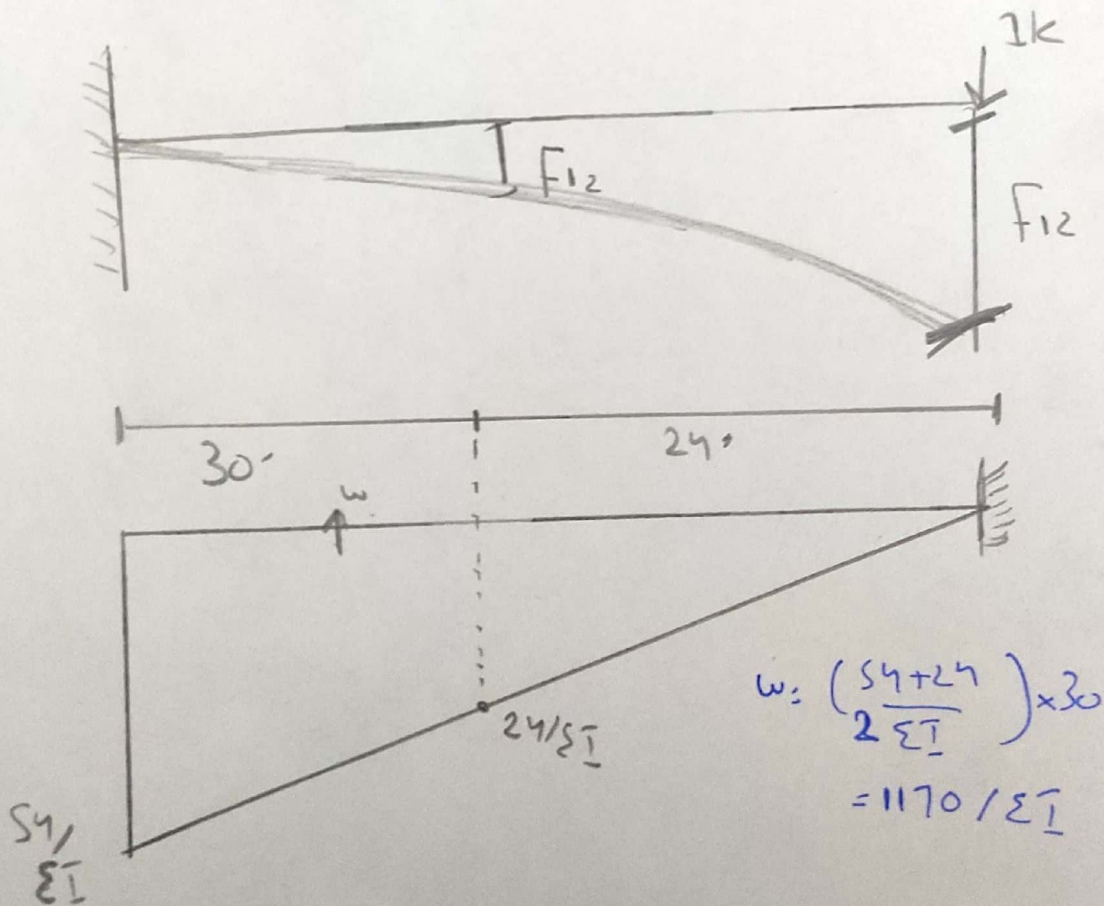


So,

$$F_{11} = \frac{450 (20)}{EI} = 9000/EI$$

$$F_{21} = \frac{450 (90+24)}{EI} = \frac{19800}{EI}$$

Now Apply Unit Load On AR2



Now the Distance

5

$$m = \frac{L}{3} \left[\frac{b + 2(a)}{a + b} \right]$$

$$= \frac{30}{3} \left[\frac{24 + 2(54)}{54 + 24} \right] = 16.92'$$

$$\Rightarrow F_{12} = \frac{1170}{EI} \times 16.92 = \frac{19796.4}{EI}$$

$$\Rightarrow F_{22} = \frac{1170}{EI} \times (16.92 + 24) = \frac{47876.4}{EI}$$

Hence

$$F_{2 \times 2} = \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix} \frac{1}{EI}$$

STEP #4 Compute the values of AR

$$[D_{RS}] = [D_{RL}] + [F] \times [AR]$$

$$[AR] = [D_{RS} - D_{RL}] \times [F]^{-1}$$

$$[F]^{-1} = \frac{1}{|F|} \times \text{Adj } F$$

$$= \frac{1}{|F|}$$

$$\begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix}$$

$$\times \text{Adj} \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix}$$

$$\Rightarrow |F| = (9000 \times 47876.4 - 19796.4 \times 19800)$$
$$(430887600 - 391968720)$$

$$|F| = 38918880$$

$$\Rightarrow \text{Adj } A = \begin{bmatrix} 47876.4 \\ -19800 \end{bmatrix}$$

$$\begin{bmatrix} -19796.4 \\ 9000 \end{bmatrix}$$

(6)

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 & -729000 \\ 0 & -1895400 \end{bmatrix} \cdot \frac{1}{\Sigma I} \times \frac{1}{38918880} \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$= \begin{bmatrix} -729000 \\ -1895400 \end{bmatrix} \cdot \frac{1}{\Sigma I} \times \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$38918880$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 66.193 \\ -67.508 \end{bmatrix}$$

FORCE METHOD

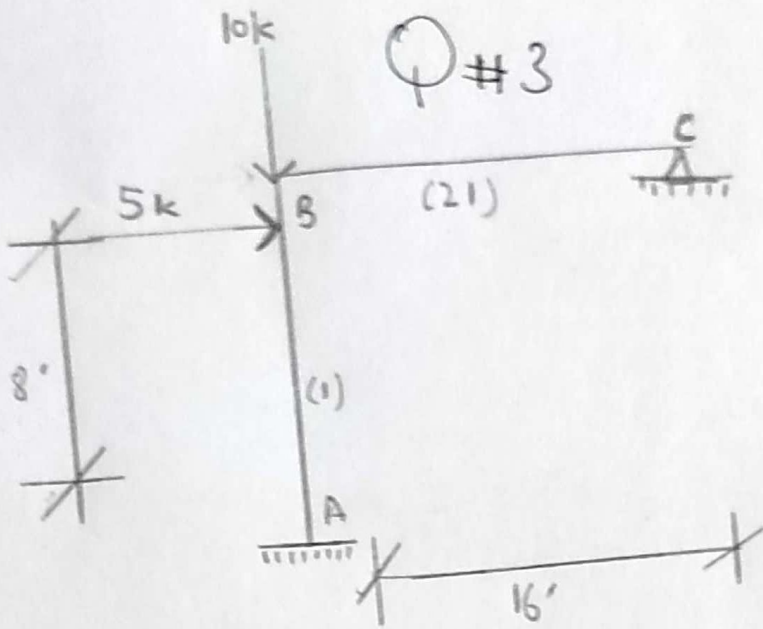
- 1) $D_s < D_k$
- 2) Forces are redundant or unknowns
- 3) Starts with Equilibrium or forces.
- 4) Forces found by Compatibility Equations of displacements
5. No of Redundants = D_s
6. Not suitable for Computer.

DISPLACEMENT METHOD

- $D_s > D_k$
- Displacements are redundant or unknowns.
- Starts with Compatible deformations.
- Displacements found by equilibrium equations of forces.
- No of Redundants = D_k
- Not suitable for trusses

Stiffness Method also Called Displacement Method is more suitable for Structure Analysis matrix approach, as it is a primary method used in matrix analysis. The main advantage of this method over flexibility method is that it is conducive to Computer programming. Once the Analytical Model of the Structure has been defined, No further engineering decisions are required in the stiffness method in order to Carry out the Analysis.

8



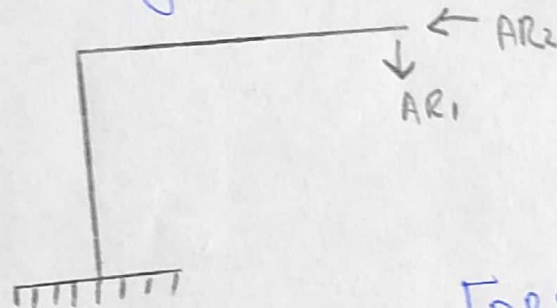
$E = \text{Constant}$
 $I_C = I$
 $I_B = 2I$

Sol:-

Total Statical Indeterminacy
 $\Rightarrow R - 3 = 5 - 3 = 2^{\circ}$

Step # 1

Identify Redundant Actions



$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step # 2

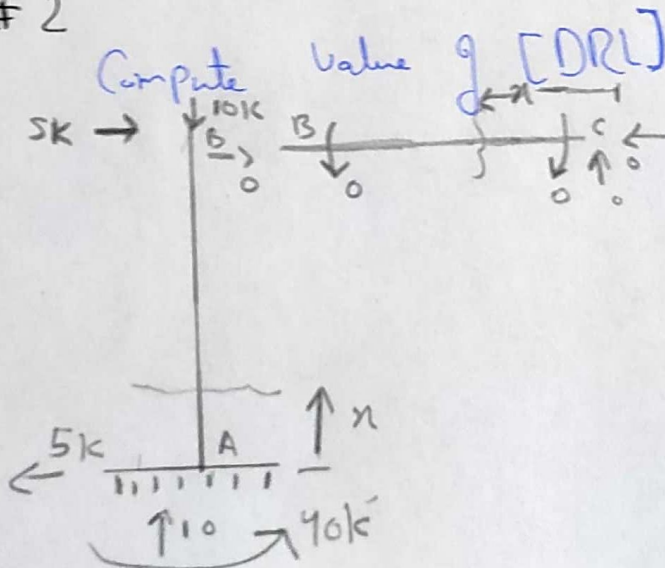


Fig: AML values (M-values)

Step # 3:-

[F] or [AMR]

(a)

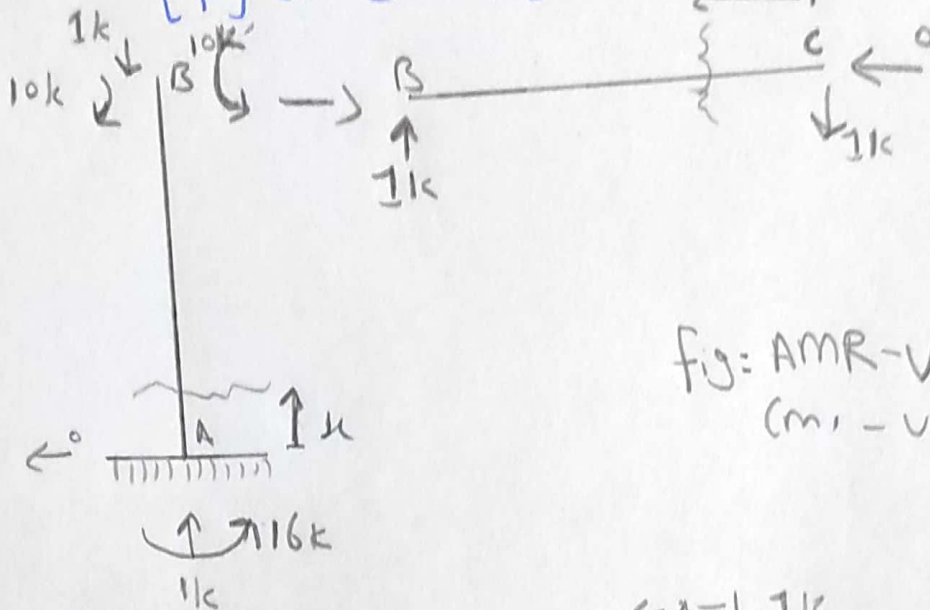


Fig: AMR-Values (m₁-values)

(b)

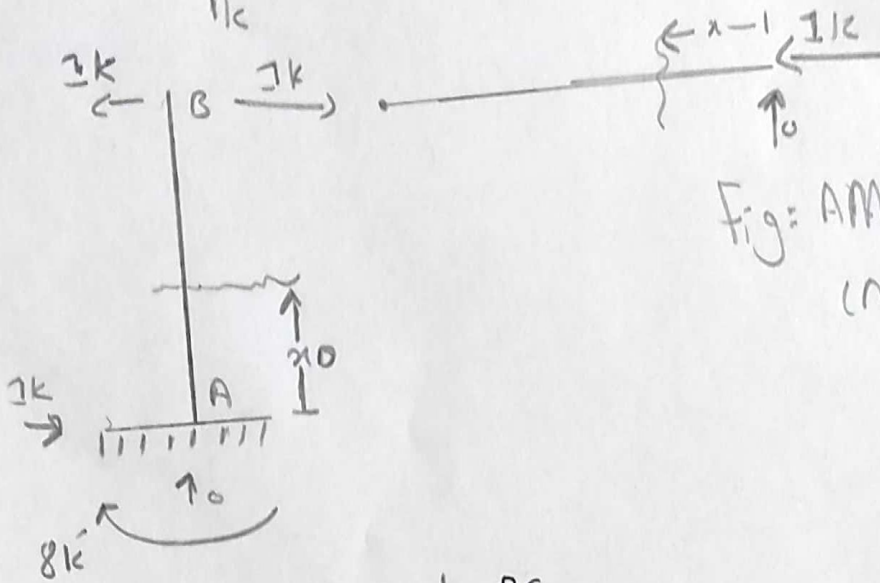


Fig: AMR values (m₂ values)

Member	AB	BC
Origin	A	C
Limits	0-8	0-16
I	I	2I
M	5x-40	0
m ₁	-16	x
m ₂	8-x	0

For finding Values of DRL's:-

$$DRL_1 = \int_0^8 \frac{M_{AB} \cdot M_1(AB)}{EI} dx + \int_0^{16} \frac{M_{BC} \cdot M_2(BC)}{EI} dx$$

$$= \int_0^8 \frac{(8x-70)(-16) dx}{EI} + \int_0^{16} \frac{0 \cdot x}{EI(2I)} dx$$

$$DRL_1 = \frac{2560}{EI}$$

$$DRL_2 = -\frac{853.33}{EI}$$

Compute Flexibility Matrix:-

$$F_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$$\rightarrow F_{11} = \int_0^8 \frac{m_1^2(AB)}{EI} dx + \int_0^{16} \frac{m_1^2(BC)}{EI} dx = \int_0^8 \frac{(-16)^2}{EI} dx + \int_0^{16} \frac{x^2}{EI(2I)} dx$$

$$F_{11} = \frac{2730.67}{EI}$$

$$F_{12} = F_{21} = \int_0^8 m_1(AB) \cdot m_2(AB) dx + \int_0^{16} m_1(BC) \cdot m_2(BC) dx$$

$$= \int_0^8 \frac{(-16)(8-x) dx}{EI} + \int_0^{16} \frac{(x)(0) dx}{2EI}$$

$$F_{12} = F_{21} = -\frac{512}{EI}$$

(11)

$$F_{22} = \int_0^8 (m_2)^2_{AB} dx + \int_0^{16} (m_2)^2_{BC} dx$$

$$= \int_0^8 \frac{(8-x)^2}{EI} dx + \int_0^{16} \frac{0^2}{2EI} dx$$

$$F_{22} = 170.67$$

As we know

$$[DRS] = [DRL] + [AR] \times [F]$$

$$\Rightarrow [AR] = \frac{[DRS] - [DRL]}{[F]}$$

$$\Rightarrow [AR] = [F]^{-1} \times [DRS - DRL]$$

$$= \begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 - 2560 \\ 0 + 853.33 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -0.00005 \\ 4.997 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$