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PAPER:- Differential equations.

Batch :- 2014.

Q1

$$\frac{dy}{dt} = e^{y-t}\sec(y)(1+t^2) \qquad y(0)=0$$

first we separate and arrange the equation so according to that we will integrate

As given $\frac{dy}{dt} = e^{y-t}\sec(y)(1+t^2)$

$\Rightarrow \frac{dy}{dt} = e^{y}\cdot e^{-t}\sec(y)(1+t^2)$

$\Rightarrow \frac{dy}{dt} = \frac{e^{y}\cdot e^{-t}}{\cos(y)}(1+t^2)$ $\qquad \therefore \cos(x) = \frac{1}{\sec(x)}$

$\Rightarrow$ Arranging

$\Rightarrow \cos(y)dy = e^{y}e^{-t}(1+t^2)dt$

$\Rightarrow e^{-y}\cos(y)dy = e^{-t}(1+t^2)dt$

Now applying integration

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$$\Rightarrow \int e^{-y}\cos(y)\,dy = \int e^{-t}(1+t^2)\,dt$$

$\therefore$ integrating RHS by parts

$$\Rightarrow \frac{e^{-y}(\sin(y)-\cos(y))}{2} = -e^{-t}(t^2+2t+3)+C \quad \text{— (A)}$$

Now we find the value of c

As initial condition was given

$$y(0)=0$$

$$\Rightarrow \frac{e^{-(0)}(\sin(0)-\cos(0))}{2} = -e^{-(0)}(0^2+2(0)+3)+C$$

$$\Rightarrow \frac{1}{2}(-1) = -(3)+C$$

$$\Rightarrow \boxed{C = \frac{5}{2}} \quad \text{— (B)}$$

put (B) in (A)

$$\text{(A)} \Rightarrow \boxed{\frac{e^{-y}}{2}(\sin(y)-(\cos(y)) = -e^{-t}(t^2+2t+3)+\frac{5}{2}}$$

Hence solved.

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Q_2 $(\sqrt{x+y}+\sqrt{x-y})dx-(\sqrt{x+y}-\sqrt{x-y})dy=0$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \quad \text{--- (1)}$$

let

$$y=Vx$$

$$\Rightarrow \frac{dy}{dx}=V+x\frac{dV}{dx}$$

eq (1) $\Rightarrow$

$$V+x\frac{dV}{dx}=\frac{\sqrt{x+Vx}+\sqrt{x-Vx}}{\sqrt{x+Vx}-\sqrt{x-Vx}}$$

$$V+x\frac{dV}{dx}=\frac{\sqrt{1+V}+\sqrt{1-V}}{\sqrt{1+V}-\sqrt{1-V}}$$

$$\Rightarrow V+x\frac{dV}{dx}=\frac{\sqrt{1+V}+\sqrt{1-V}}{\sqrt{1+V}-\sqrt{1-V}}\times\frac{\sqrt{1+V}+\sqrt{1-V}}{\sqrt{1+V}+\sqrt{1-V}}$$

solving

$$\Rightarrow V+x\frac{dV}{dx}=\frac{1+\sqrt{1-V^2}}{V}$$

$$\Rightarrow x\frac{dV}{dx}=\frac{1+\sqrt{1-V^2}}{V}-V$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

<u>integrate</u>

$$\int \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\Rightarrow \int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow -\ln t = \ln x + \ln c$$

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$$\Rightarrow -\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\Rightarrow \cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \cancel{\ln}(cx)^{-1}$$

$$\Rightarrow 1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$\Rightarrow 1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$\Rightarrow 1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$\Rightarrow x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$\boxed{x+\sqrt{x^2-y^2} = c_1} \quad \because \frac{1}{c} = c_1$$

Solved.

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Q3

Solution

As given

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow \frac{d^4y}{dx} + \frac{d^2y}{dx} = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow F(D)y = F(x)$$

Since $y = y_c + y_p$ —— (1)

y_c is linked with homogeneous part.

so

$$D^4 - D^2 = 0 \Rightarrow D^2(D^2+1) = 0$$

Either $D^2 = 0 \Rightarrow D = 0$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \Rightarrow D = i \text{ or } D = 0 + i$$

So Roots are real & complex

$$\Rightarrow y_c = C_1 e^{0x} + e^{0x}(C_2\cos x + C_3\sin x)$$

$$\Rightarrow y_c = C_1 + C_2\cos x + C_3\sin x$$

Now $\Rightarrow y_p = \frac{1}{f(D)} F(x)$

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$$y_p = \frac{1}{D^4+D^2}\left(3x^2+4\sin x-2\cos x\right)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4+D^2$

at $D=0 \Rightarrow f(D)=0$

so $f'(D) = 4D^3+2D$

Now for $D=0 \Rightarrow f'(D)=0$

Differentiate again.

$f''(D) = 12D+2$

So for $D=0$

$f''(0) = 12(0)+2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2\,3x^2}{12D+2} + \frac{x^2}{12D+2}\cdot 4\sin x - \frac{x^2}{12D+2}\cdot 2\cos x$$

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Now put $D=0$

$$y_p = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} + \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3x^4}{2} + 2x^2\sin x - x^2\cos x$$

So putt in eq (1)

$$y = C_1 + C_2\cos x + C_3\cos x + \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

$$\boxed{\Rightarrow y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$

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