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Section C

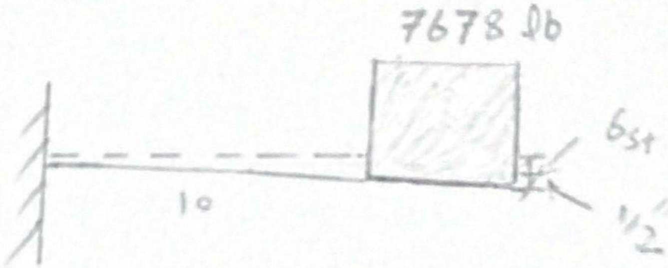
ID 7678

paper INTro. To structural Dynamics & Earthquake. Engg.

Submitted to Engr. Yaseen Mehmood.

QNo 1

Answer:



⇒ Given data:

- ⇒ $E = 29{,}000$ Ksi
- ⇒ $I = 150\ in^4$
- ⇒ Deflection due to static load = 7678 lb

⇒ Required data:

- ⇒ Determine natural time period and develop and solve the equation of motion for vibration at free end = ?
- ⇒ Develop the equation showing variation in the equivalent static force with time = ?
- ⇒ Amplitude of equivalent static force = ?

Solution

As we know that;

The general EOM for SDOF is;

$$Ku + c\dot{u} + m\ddot{u} = P(t) \longrightarrow (i)$$

In our case system is undamped $c = 0$

And undergoing free vibration $P(t) = 0$

$\Rightarrow$ Hence general EOM becomes;

$$\Rightarrow Ku + m\ddot{u} = 0 \longrightarrow (ii)$$

Now;

$$\Rightarrow K = 3EI/L^3$$

$$\Rightarrow K = \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$\boxed{K = 7.55 \text{ K/in}}$$

$$\Rightarrow \boxed{K = 7.55 \text{ K/in} = 90625 \text{ lb/ft}}$$

And;

$$m = \frac{7678}{32.2} \frac{\text{lb.sec}^2}{\text{ft}}$$

$$\Rightarrow \boxed{m = 238.44 \text{ slug}}$$

$$\Rightarrow \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{238.44}} = 19.495 \text{ rad/sec}$$

$$\Rightarrow T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.495} = 0.322 \text{ Sec}$$

Substituting the Corresponding value in eq (ii) we get;

$$\Rightarrow Ku + m\ddot{u} = 0$$

$$\Rightarrow 90625u + 238.4\ddot{u} = 0$$

Where; "K" is in lb/ft and "m" in lb sec/ft^2

Now;

$\Rightarrow$ General Solution to the EOM for undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

Where;

$$\Rightarrow u(0) = 1/2'' = \frac{1}{2\times 12} = \frac{1}{24} \text{ ft} \quad \text{and } \dot{u}(0) = 0$$

$$\Rightarrow u(t) = \frac{1}{24} \times \cos(19.495t) + 0$$

$$\Rightarrow u(t) = \frac{1}{24} \times \cos(19.495t)$$

Now Equilant static force at any time "t" is

$$\Rightarrow f_s(t) = K.u(t) = \frac{90625 \times \cos(19.495t)}{24}$$

$$\Rightarrow f_s(t) = 3776.04(\cos(19.495t))$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]} = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

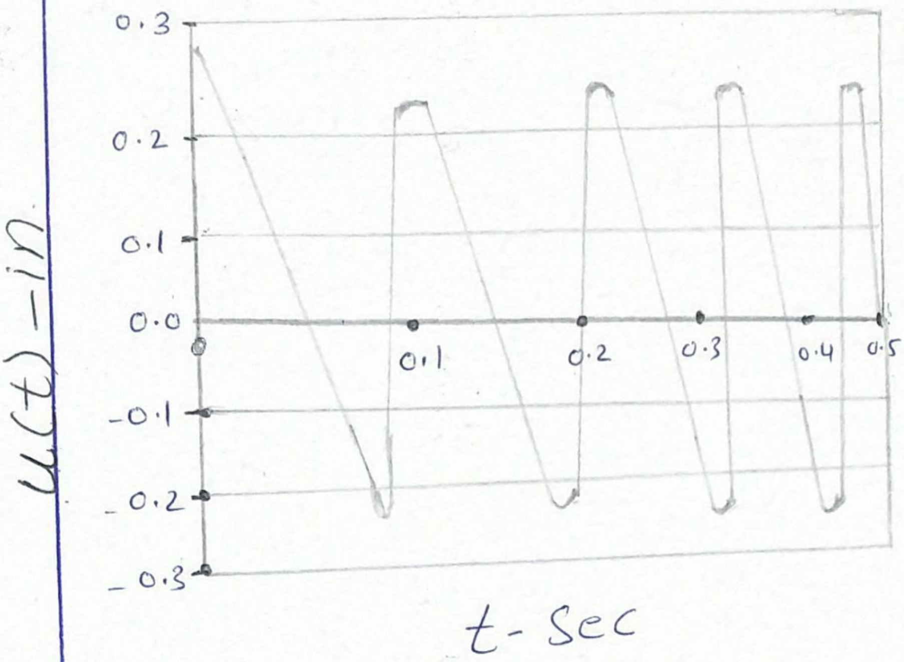
$$\boxed{u_o = \frac{1}{24} \text{ ft}}$$

Amplitude of equivalent static force, f_{so}

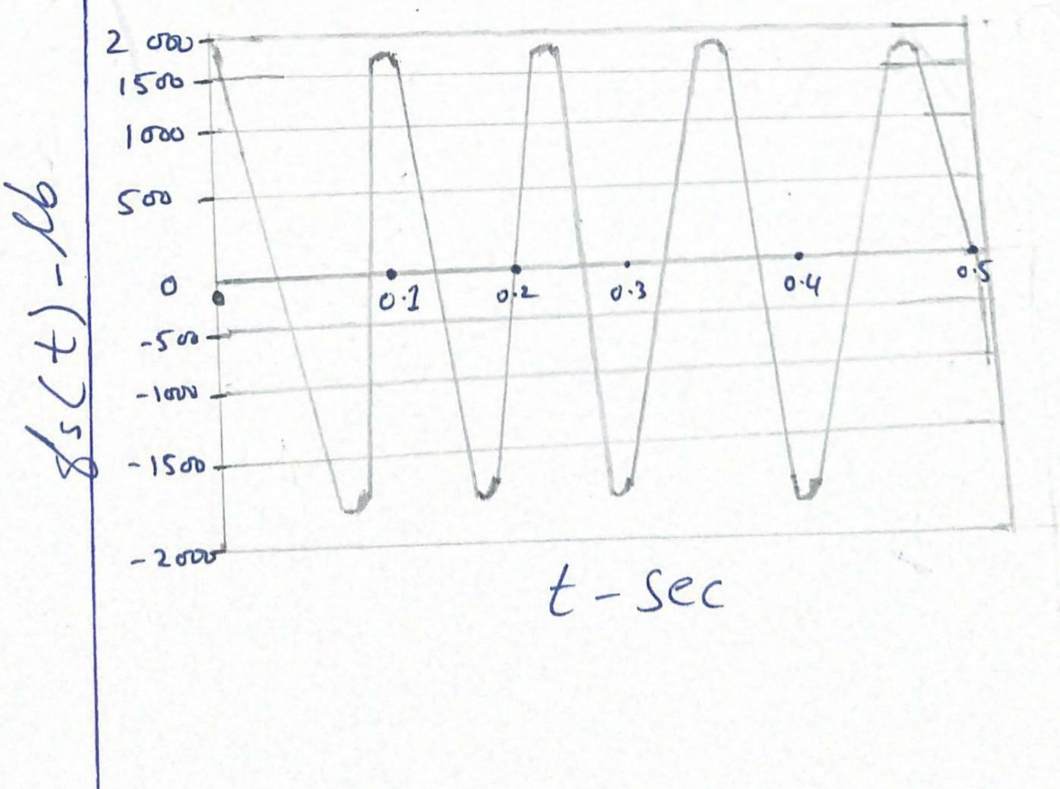
$$\Rightarrow K u_o = 90625 \times \frac{1}{24} =$$

$$\Rightarrow \boxed{K u_o = 3776.04 \text{ lb}}$$

undamped Free vibration



⇒ Vibration of displacement with time



Q No 2

Given data:

=> Damping Ratio of Reinforced Concrete with Considerable Cracking = 3–5%.

=> So we take = 3%.

=> Other data from Q No 1 we get;

Required data:

=> Develop and Solve the equation of motion for vibration at free end = ?

=> Develop an equation Showing variation in equivilant State force with time = ?

Solution:

As we know that

EOM for damped free vibration is;

=> $Ku + C\dot{u} + m\ddot{u} = 0 \longrightarrow (i)$

Now we take value of K, and m value from QNo 1 we get;

$\Rightarrow K = 90625 \text{ lb/ft}$

$\Rightarrow m = 238.44 \text{ lb.sec}^2\text{/ft}$

$\Rightarrow W_n = 19.495 \text{ rad/sec}$

As we Know that;

$\Rightarrow C = \zeta \times 2m \times W_n$

$= 0.03 \times 2 \times 238.44 \times 19.495$

$\Rightarrow \boxed{C = 278.90 \text{ lb-sec/ft}}$

Now putting value in eq ① we get

$\Rightarrow 90625\, u + 278.90\, \dot{u} + 238.44\, \ddot{u} = 0$

$\Rightarrow$ Solution to The EOM for damped free vibration is;

$$\Rightarrow u(t) = e^{-\zeta w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left[\dot{u}(0) + u(0)\zeta w_n\right]\sin(w_D t)\right]$$

Here $W_D = W_n = 19.495$ rad/sec

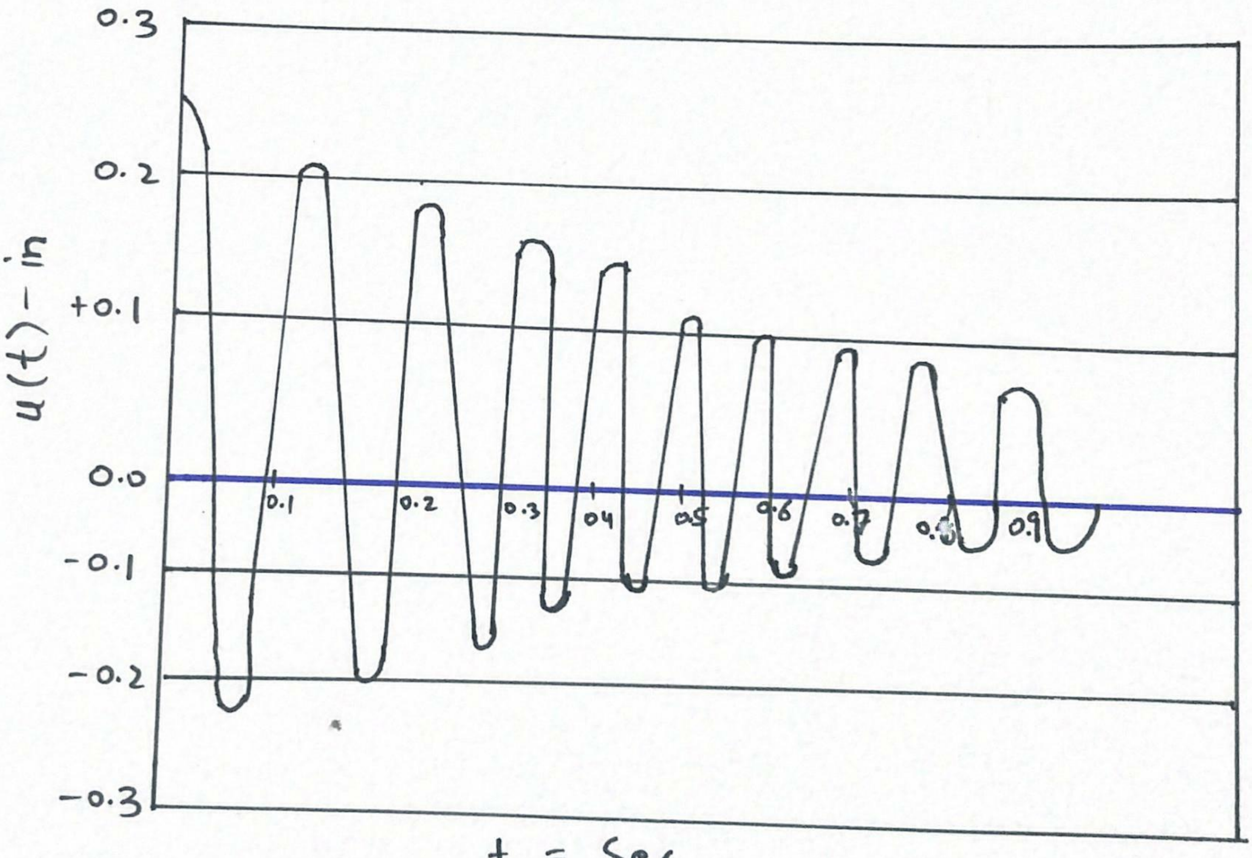
$$\Rightarrow u(t) = e^{(-0.03 \times 19.495 \times t)}\left[\frac{1}{24} \times \cos(19.495t) + \frac{1}{19.495}\left[0 + \frac{1}{24} \times 0.03 \times 19.495 \times \sin(19.495t\right.\right.$$

$$\Rightarrow u(t) = e^{-0.584}\left[0.0416 \times \cos(19.495t) + 0.051(0.024 . \sin 19.49 t)\right]$$

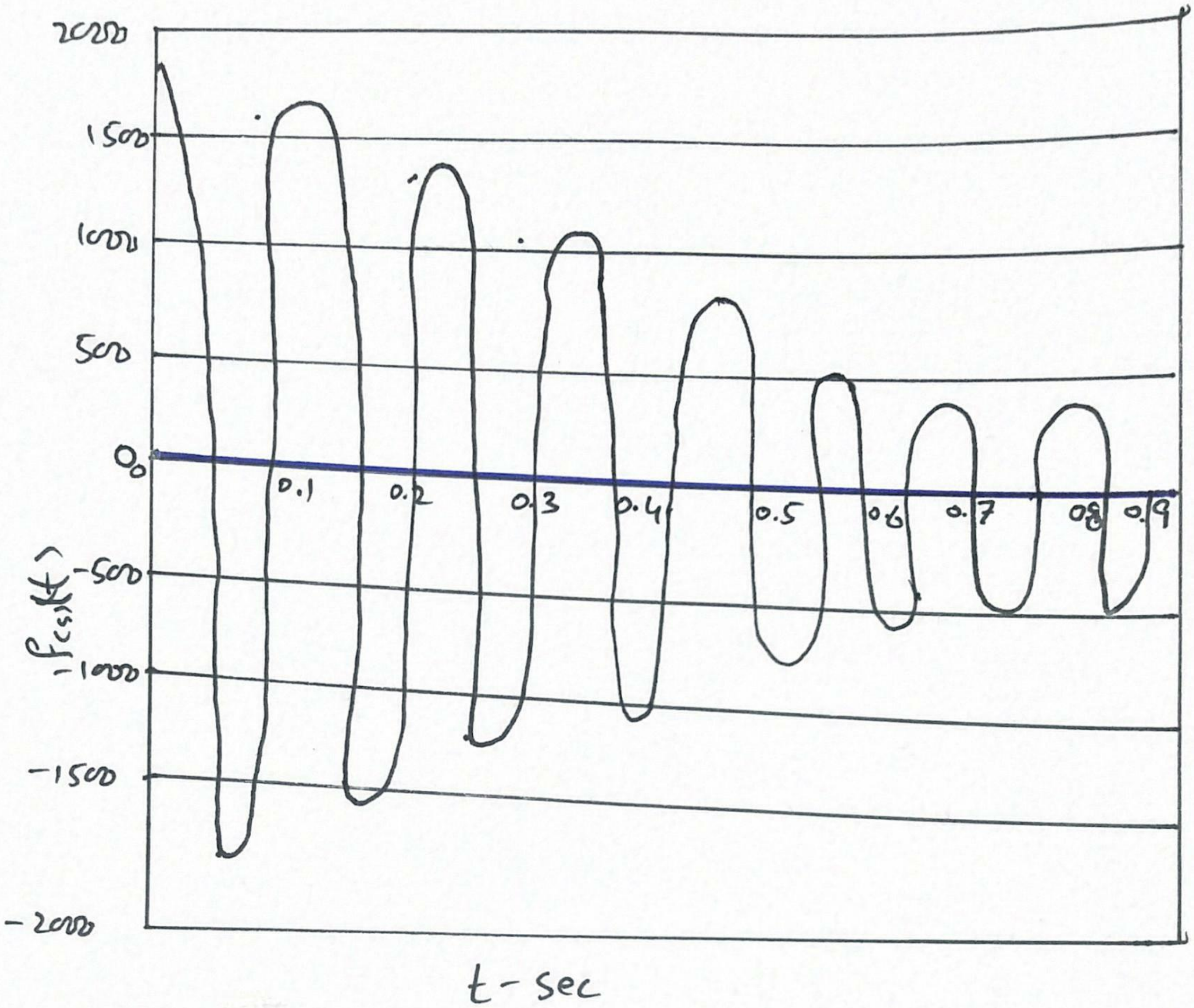
$$\Rightarrow f_s(t) = K.u(t) = 90625 \times u(t)$$

$$\Rightarrow f_s(t) = e^{-0.584}\left[3770 \cos(19.49 t) + 110.925 \sin(19.49 t)\right]$$

Graph:



variation of displacement with time.



variation of Equuvilant static force with time.

End

Q No 3

Given data:

⇒ Applied force Through Cable = 60 Kips

⇒ Displace tank = $\frac{7678}{1000}$ = 7.678"

⇒ No. of cycles = 7 cycles

⇒ Completing cycles in = 3.57 sec

⇒ The Amplitude of displacement = 2.286 cm

Required data:

(a) Damping Ratio = ?

(b) Natural period of undamped vibration = ?

(c) Stiffness of structure = ?

(d) Weight of tank = ?

(e) Damping co-efficient = ?

(f) Number of cycles to reduce the displacement amplitude to 0.5"

Solution:

As; $u_1 = 7.678"$ [∵ 2.286 cm = 0.9"]

⇒ After $j = 7$, $u_{j+1} = u_8 = 0.9"$

$\Rightarrow$ (a) ξ = Damping Ratio = ?

$$j = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow 7 = \frac{1}{2\pi\xi} \ln\left[\frac{7.678}{0.9}\right]$$

$$\Rightarrow \xi = \frac{1}{2\pi \times 7} \ln\left[\frac{7.678}{0.9}\right] = 0.0487$$

$$\boxed{\xi = 4.87\,\%}$$ Answer.

$\Rightarrow$ (b) T_n = ?

7 cycles of vibration are completed in $\doteq$ 3.57 sec

$\Rightarrow$ T_D = Time Required to complete one cycle = $\frac{3.57}{7} = 0.51$

$T_D = 0.51$ sec

$$\Rightarrow \omega_D = \omega_n \sqrt{1-\xi^2}$$

$$\Rightarrow \frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n \sqrt{1-\xi^2}}$$

*xing by $\frac{1}{2\pi}$ B.S

$\because T_D = \frac{2\pi}{\omega_D}$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{1-\xi^2}}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2} = 0.51 \times \sqrt{1-(0.0487)^2}$$

$$\Rightarrow \boxed{T_n = 0.5093 = 0.51 \text{ Sec}}$$ Answer.

=> (c) $K = ?$

$$\Rightarrow K = \frac{60 \times \cos 60^\circ}{7.678} = 3.90 \text{ K/in}$$

$$\boxed{K = 46800 \text{ lb/ft}}$$ Answer;

(d) weight of tank = W = ?

$$\Rightarrow \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{W/g}} = \frac{K \times g}{W} \qquad \because \omega_n = \frac{2\pi}{T_n}$$

$$\Rightarrow \omega_n^2 = \frac{K \times g}{W} \Rightarrow W = Kg \Big/ \frac{4\pi^2}{T_n}$$

$$\Rightarrow W = \frac{46800 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$$= 9928.47 \text{ lb}$$

$$\Rightarrow \boxed{W = 9.928 \text{ K}}$$ Answer;

(e) Damping Co-efficient = ?

$C = ?$

We know that;

$$\Rightarrow \zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \zeta \times 2m\omega_n = \zeta \times 2m\left(\frac{2\pi}{T_n}\right)$$

$$= \frac{0.0487 \times 4\pi \times \frac{9928.47}{32.2}}{0.51}$$

$$\Rightarrow \boxed{c = 369.99 \text{ lb-Sec/ft}}$$ Answer.

(f) No. of cycle to reduce displacement amplitude to 0.5" = ?

As we Know;

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\pi(0.0487)} \times \ln\left[\frac{7.678}{0.5}\right]$$

$$j = 8.926 \text{ or } 9 \text{ cycles}$$