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Subject	*Advance fluid*
Assignment	*Final term*
Semester	*8th*
Submitted to	*Engr Abdul Waheed*

Q No 1 ①

(a) Drag:

A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion b/w body and fluid these forces are termed as drag and lift. if the forces parallel to the motion then it is termed as drag force.

⇒ There are two Components.

① Pressure Drag (FP):

It is equal to integrated of Components in direction of motion of all pressure forces extended on Surface of body.

$$\cancel{FP = CP \frac{\rho v^2}{2} A}$$

$$FP = CP \int \frac{v^2}{2} A$$

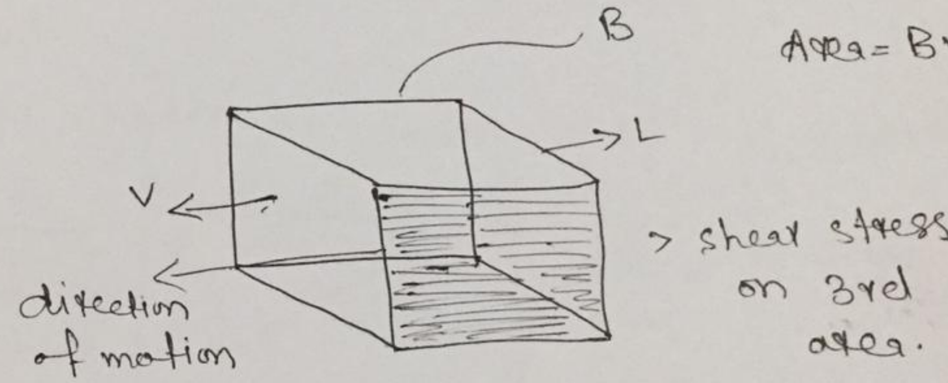
where CP depends on shape.

(2) Friction Drag (Ff)

it is equal to integration of components of shear stress along surface of body in direction of motion.

$$F_f = \underbrace{c_f \int \frac{v^2}{2}}_{\text{shear stress}} BL$$

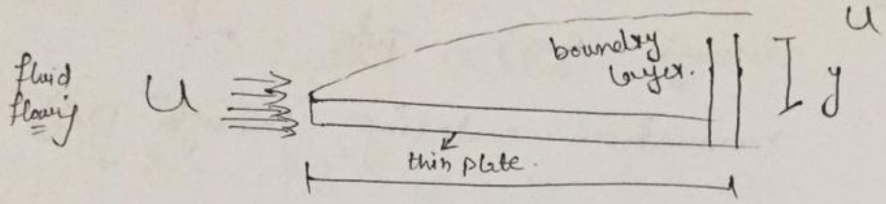
Fig:.



Area = B√L

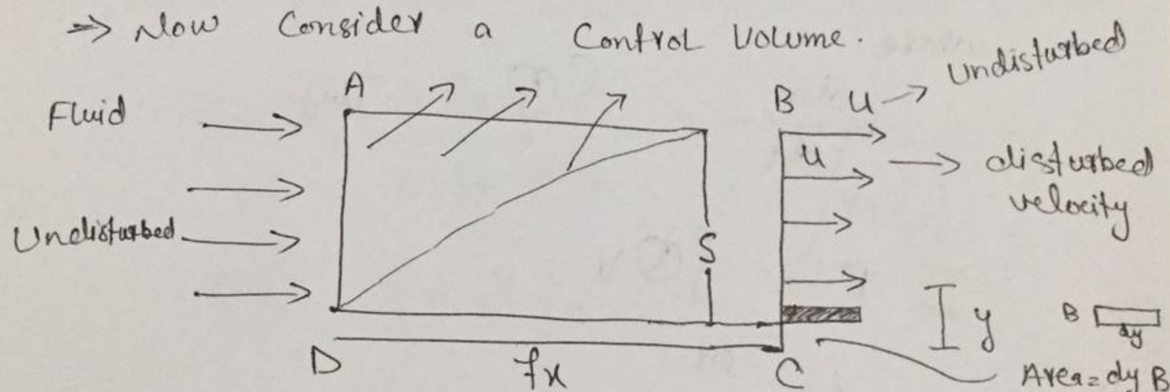
③

⇒ Friction Drag of Boundry Layer



⇒ Fig shows growth of boundry layer along one site of smoth plate inside the fluid.

⇒ Now Consider a Control Volume.



⇒ where δ is thickness of boundry layer and U is Undisturbed velocity

Thus $-F_x$ = drag = (rate in momentum in x- direction)

⇒ Leaving through BC + rate of momentum through AB) − rate of momentum entering through DA)

$\Delta P = P_{out} - P_{in}$

Thus according to momentum

$$\Sigma F = \frac{d}{dt}(P) = \frac{d\, mv}{dt}$$

where

$$\frac{dm}{dt} = \rho Q$$ Thus

$$F = \rho Q V$$

or

$$F = \rho A \cdot V \cdot V$$

$$F = \rho A V^2$$

$DA \longrightarrow \rho U (UBS)$

$BC \longrightarrow \rho B \int_0^s u^2 \cdot dy$

$AB \longrightarrow \rho U \left(UB\delta - B\int_0^\delta u \cdot dy \right)$

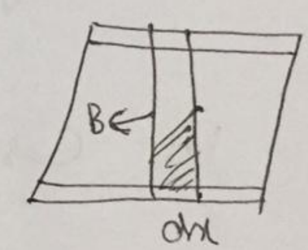
Putting value

$$F_x = \rho B \int_0^\delta u\,(U-u)\,dy$$

$$F_x = \rho B U^2 \delta \alpha$$ where α is function of boundary layer.

Now to find local wall shear stress

$$\tau_0 = \frac{d f_x}{B \cdot dx - \text{area}}$$



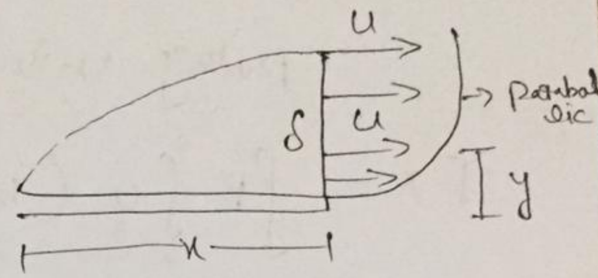
$$F_x = \rho B U^2 \delta \alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$ in general

equation of shear stress.

→ Laminar boundary Layer :-

$$\frac{u}{V} = F\left(\frac{y}{\delta}\right)$$



Assume

$$\eta = \frac{y}{\delta} \quad \text{or} \quad y = \eta\delta$$

Thus

$$\frac{u}{V} = f(\eta) \quad \text{or} \quad u = Uf(\eta)$$

In Case of laminar flow

$$\tau_0 = \mu\left(\frac{du}{dy}\right)$$

$$= \frac{\mu}{\delta}\left(\frac{du}{d\eta}\right) = \frac{\mu U}{\delta}\left[\frac{df}{d\eta}(\eta)\right]$$

Solving the Eq.

$$\tau_0 = \frac{\mu U B}{\delta} \rightarrow (1)$$

As general Equation is $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$,

Equating both equation.

$$\frac{\mu U B}{\delta} = \rho V^2 \alpha \frac{d\delta}{dx}$$

or

$$\delta \, d\delta = \frac{\mu B}{\rho U \alpha} dx$$

Integrating the equation.

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho \mu \alpha.} x + C$$

Now at $x = 0$, $\delta = 0$ Thus $C = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho \mu \alpha} x.$$

or

$$\delta = \sqrt{\frac{2\mu B}{\rho U \alpha} x} \quad \text{or} \quad \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

x'ing and ÷'ing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu \cancel{x}}{\rho U}} \cdot \frac{x}{\sqrt{\cancel{x}} \cdot \sqrt{x}}$$

where $\alpha = 0.135$

$B = 1.63$

$R_x = \frac{\rho U x}{\mu}$

$$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now

$$\tau_0 = \frac{\mu U B}{\delta}$$

Thus putting value

$$\boxed{\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}}$$

where R_x is local Reynold number.

Now

$$F_g = B \int_0^l \frac{\tau_0 \, dx}{\text{stress}}$$

Putting values

$$F_g = 0.664 \, B \sqrt{\rho \mu L V^3}$$

As general equation is

$$F_f = C_f \, \rho \frac{V^2}{2} BL \longrightarrow$$ Equating both equation

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L V}} = \frac{1.328}{\sqrt{R}}$$

———— x ————

TURBULANT BOUNDARY LAYER:

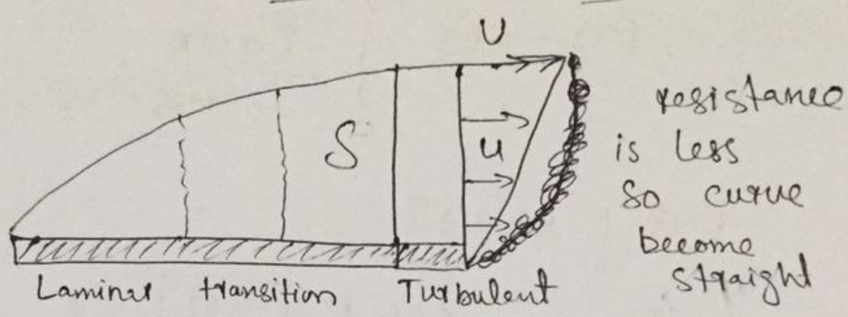


Fig show the velocity distribution in turbulent boundary layer shows a much steeper gradient near wall and flatter through out reming layer.

⇒ To shear stress is greater in turbulent than in liminar layer.

As we have.

$$\tau_0 = \frac{f \rho v^2}{8}$$

where v denotes average velocity of pipe.

→ now we have obtained an approximate relation b/w V and U by using pipe factor Equation of

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{f}}$$

Using fraction factor of 0.028 from chart which is middle critical value.

So

$$\boxed{U = 1.235V}$$

Now we have

$$\tau_0 = \frac{f \rho V^2}{8}$$

As we know

$$f = \frac{0.316}{R^{0.25}}$$

Thus

$$\tau_0 = \frac{0.316}{\left(\frac{DV}{\nu}\right)^{1/4}} \rho \frac{V^2}{8}$$

where

$$V = \frac{U}{1.235} \quad \text{thus}$$

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{V}{1.235}\right)^2$$

$$\in D = 2\delta$$

Thus

$$\tau_0 = \frac{0.0238\, U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}}$$

As we have

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Equating both and Integrating For boundary Condition of $x = 0$, $\delta = 0$

Thus

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5}\left(\frac{\nu}{Ux}\right)^{1/5} x.$$

For $\alpha = 0.0972$

$$\boxed{\frac{\delta}{x} = \frac{0.377}{(Rx)^{1/5}}}$$

Putting values in equation.

$$\tau_0 = 0.0587\, \rho \frac{U^2}{2}\left(\frac{U}{\nu x}\right)^{1/5}$$

Now

$$F_f = B\int_0^2 \tau_0\, dx$$

$$F_f = 0.0735\, \rho \frac{U^2}{2}\left(\frac{\nu}{UL}\right) BL$$

As $F_f = C_f\, \rho \frac{V^2}{2} BL$.

Equating Both:

$$\boxed{C_f = \frac{0.0735}{R^{1/5}}}$$

R is less than 10^7 fo

$500{,}000 < R < 10^7$

For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.58}}}$$

Ans

X

End

Q No1
Part (B)

As specific Energy

$$E = y + \frac{V^2}{2g}$$

the flow Q per unit width b can be expressed as

$$q = \frac{Q}{b}$$

Now average velocity will be

$$V = Q/A = \frac{qb}{by} = \frac{q}{y}$$

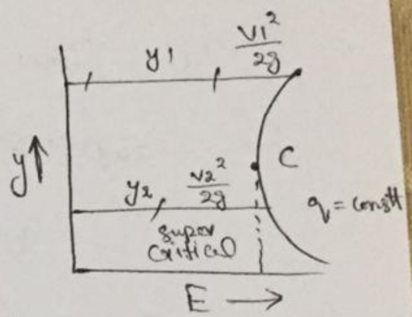
Thus

$$E = y + \frac{V^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E-y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right) \quad \text{or} \quad (E-y)y^2 = \frac{q^2}{2g}$$

Thus plot of E vs Y will be parabolic. For particular q, there will be two kind of possible values of y, for a given E.

The Equation is cubic with three roots, with third root being negative point c represents dividing point b/w two regime of flow



Thus for given q, & value of E is minimum & Flow at that Point is critical flow. Depth of flow at that point is critical depth y_c & velocity at that point is critical velocity V_c.

Thus

③

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum specific energy $\frac{dE}{dy} = 0$

Thus

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right) = 0$$

$$\Rightarrow \frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\frac{q^2}{g} = y^3 \Rightarrow \left(\frac{q^2}{g}\right)^{1/3} = y_{cr}$$

Now

$$q^2 = gy^3$$

&

$$q = vy \Rightarrow v^2y^2 = gy^3$$

or

$$v^2 = gy_{cr}$$

or

$$V_{cr} = \sqrt{g\,y_{cr}}.$$

Q2 Given

Water Flows at rate $Q = 3.5\ m^3/s$

Bed Slope, $S_o = 0.0008$

$n = 0.0219$

width of bed is student ID = 7681

Required:

Depth of rectangular channel = ?

Critical Depth $y_c = ?$

Critical velocity $V_c = ?$

Flow is critical or sub critical = ?

Solution:

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2}\right) A \longrightarrow (1)$$

$$\text{Area} = 7.681 \times d$$

$$\text{Parameter} = d + 7.681 + d$$

(2)

Parameter = $d + 7.681 + d$

hydraulic Radius = $R_n = \dfrac{\text{Area}}{\text{Parameter}}$

$$= \frac{7.681 \times d}{2d + 7.681}$$

We know that

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2}\right) A$$

put the value

$$3.5 = \left(\frac{1}{0.0219} \times \left(\frac{7.681 \times d}{2d + 7.681}\right)^{2/3} \times \left(0.0008\right)^{1/2}\right) \times 7.681d$$

$$\frac{3.5 \times 0.0219}{\sqrt{0.0008}} = \left(\frac{7.681 \times d}{2d + 7.681}\right)^{2/3} \times 7.681\,d$$

$$(2.59)^{3/2} = \frac{7.681\,d}{2d + 7.681} \times 7.681\,d$$

3

$$(4.461)(2d + 7.681) = 58.99d^2$$

$$8.92d + 34.21 = 58.99d^2$$

$$58.99d^2 - 8.92d - 34.21 = 0$$

$$\boxed{d = 0.840m}$$

So the depth of channel is 0.840m

As $q = \frac{Q}{b}$

$$q = \frac{3.5}{7.681} = 0.455$$

For critical depth

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$y_{cr} = \left(\frac{(0.455)^2}{(9.81)}\right)^{1/3}$$

(4)

$y_{cr} = 0.276\,m$

Now critical velocity

$$V_{cr} = \sqrt{y_{cr} \times g}$$

$$V_{cr} = \sqrt{(9.81)(0.276)}$$

$$V_{cr} = 1.645\ m/s$$

$$V = \frac{Q}{A}$$

$$V = \frac{3.5}{7.681 \times 0.840}$$

$$V = 0.542\ m/s$$

$y = 0.840 m$, $y_{cr} = 0.276 m$

$V = 0.542 m/s$, $V_{cr} = 1.645 m/s$

As $y > y_{cr}$ and $V < V_{cr}$

So the flow is sub critical

x

①

Q3 Given DATA:

width of Smooth plate $B = 200$ mm

Length of Smooth plate $L = 800$ mm

$= 0.8$m

Oil with specific gravity ; $S = 0.89$

Undisturbed velocity , $u = 5$ m/sec

Kinematic viscosity $\nu = 0.93 \times 10^{-4}$ m^2/s

Required DATA:

Friction drag on one side of a Smooth plate , $F_f = ?$

SoL: Check the flow.

As $\nu = 0.93 \times 10^{-4}$ m^2/s .

$$R = \frac{Lu}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$R = 43010.75 < 500,000$

Thus flow is Laminar.

Now

$$Cf = \frac{1.328}{\sqrt{R}} \Rightarrow \frac{1.328}{\sqrt{43010.75}}$$

$$Cf = 6.403 \times 10^{-3}$$

$$Cf = 0.0064$$

$$\Rightarrow Ff = Cf \, \rho \, \frac{V^2}{2} \, BL$$

$$= (0.0064)(\text{Soil} \times \gamma_{water}) \times \frac{5^2}{2} \times 0.2 \times 0.8$$

$$= \left((0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)\right)$$

$$\boxed{Ff = 11.392 \ N}$$

Ans