SUBMITTED BY : MUDASIR

SUBMITTED to : Maam SHUMAILA

ID : NO : 7755

Subject : Differential equation

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Question NO : 01

Solution:-

$$\frac{dy}{dt} = e^{y-t}\, Sec(y)\,(1+t^2) \qquad y(0)=0$$

As we know that So $x=0$, $y=0$

$$dy = e^{y}\cdot e^{-t}\, Sec(y)\,(1+t^2)\,dt$$

$$\frac{1}{e^{y}\,Sec(y)}\,dy = (1+t^2)\,e^{t}\,dt$$

As $\cos(y) = \frac{1}{Sec(y)}$

$$\int e^{-y}\cos y\,dy = \int (1+t^2)\,e^{-t}\,dt$$

Using Integration by parts

$$e^{-y}\int \cos y\,dx - \int\left(\cos y \cdot \frac{d}{dy}e^{-y}\right) = (1+t^2)\int e^{-t} - \int\left(\int e^{-t}\frac{d}{dt}(1+t^2)\right) \rightarrow (1)$$

NOW L.H.S

$$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \cdot \frac{d}{dy}e^{-y}\right)$$

$$e^{-y}\sin y - \int \sin y \cdot e^{-y}(-1)$$

$$e^{-y}\sin + \int \sin y \cdot e^{-y}$$

$e^{-y}\sin y + \int (e^{-y}\sin y)$

Again Using Integration by parts

$e^{-y}\sin y + e^{-y}(-\cos y) - \int \left(\int \sin y \frac{d}{dy} e^{-y}\right)$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int \left(-\cos y \frac{e^{-y}}{-1}\right)$

$e^{-y}\sin y - e^{-y}\cos y - \int (\cos y\, e^{-y})$

Since $\int (\cos y e^{-y}) = L.H.S$

So it is again Same to the first one So L.H.S will become

$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$

$2\, L.H.S = e^{-y}(\sin y - \cos y)$

$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S

$\int (1+t^2)\, e^{-t}\, dt$

$1+t^2 \int e^{-t} - \int\left(\int e^{-t} \cdot \frac{d}{dt}(1+t^2)\right)$

$-(1+t^2)e^{-t} - \int -e^{-t}(2t)$

$(1+t^2)e^{-t} + \int ((2t)e^{-t})$

Again Using Integration by Parts

$$-(1+t^2)e^{-t}+2t\int e^{-t}-\int\left(\int e^{-t}\frac{d}{dt}2t\right)$$

$$-(1+t^2)e^{-t}+\left(-2te^{-t}-\int(-e^{-t}2)\right)$$

$$-(1+t^2)e^{-t}+\left(-2te^{-t}+\int(2e^{-t})\right)$$

$$-(1+t^2)e^{-t}+\left(-2te^{-t}-2e^{-t}\right)+c$$

$$-(1+t^2)e^{-t}+2te^{-t}-2e^{-t}+c$$

$$-(1+t^2)e^{-t}-2te^{-c}-2e^{-t}+c$$

$$-e^{-t}-e^{-t}t^2-2te^{-t}-2e^{-t}+c$$

$$-(t^2+2t+3)e^{-t}+c=R.H.S$$

NOW take $L.H.S=R.H.S$

$$\frac{e^{-y}\sin y-\cos y}{2}=-(t^2+2t+3)e^{-t}+c$$

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We know that

$t = 0$ & $y = 0$

Put it above

$\frac{1}{2}(0-1) = 0 - 3 + c$

$c = 5/2$

Put value of c

$$\boxed{e^{-y}(\sin y - \cos y) = -(x^2 + 2t + 3)e^{-t} + 5/2}$$

Answer.

Question No # 2

Solution:-

$$\left(\sqrt{x+y}+\sqrt{x+y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right)dy=0$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$$

This is Homogeneous Differential equation in "x" and "y" to solve this

$$y=vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Thus eq (1) become

$$v+x\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{1+v+\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}\ 1-\cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1+\sqrt{1-v^2}\right)}{\cancel{2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

Taking Integral on Both Sides

$$\int \frac{v\,dv}{\sqrt{1-v}\left(1+\sqrt{1-v^2}\right)} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}\left(1-v^2\right)^{1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln C$$

$$-\ln\left(1+\sqrt{1-v^2}\right. = \ln Cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln Cx$$

$$\ln\left(1+\sqrt{1-v^2}\right. = \ln (Cx)^{-1}$$

$$\cancel{\ln\left(1+\sqrt{1-v^2}\right) = \ln (Cx)^{-1}}$$

$$1+\sqrt{1-v^2} = \frac{1}{Cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{Cx}$$

$$x + \sqrt{x^2 - y^2} = 1/c$$

$$x + \sqrt{x^2 - y^2} = c_1 \quad \because 1/c = c_1$$

Which is a Required Result Solution.

Answer

Question NO # 03

Solution:-

$$(D^4+D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$① \Rightarrow f(D)y = f(x)$$

As it is non Homogeneous linear equation so solution will be

$$y = y_c + y_p \longrightarrow (i)$$

Complementary solution y_c

$$D^4+D^2=0 \Rightarrow D^2(D^2+1)=0$$

Either $D^2=0 \Rightarrow D=0$

$$D^2+1=0 \Rightarrow D^2=1$$

$$D=\sqrt{-1} \Rightarrow D=i \quad \text{or} \quad D=0+i$$

Roots are Real and Complex

$$y_c = c_1e^{0x} + e^{0x}(c_2\cos x + c_3\sin x)$$

$$y_c = c_1 + c_2 \cos x + c_3 \sin x$$

$$y_p = \frac{1}{f(D)} F(x)$$

$$y_p = \frac{1}{D^4 + D^2}\left(3x^2 + 4\sin x - 2\cos x\right)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$$f(D) = D^4 + D^2$$

at $D = 0 \Rightarrow f(0) = 0$

So $f'(D) = 4D^3 + 2D$

Now also for $D = 0 \Rightarrow f'(D) = 0$

again differentiating

$$f''(D) = 12D + 2$$

So for $D = 0$

$$f''(0) = 12(0) + 2 = 3$$

So replacing $\frac{1}{f(D)}$ with $\frac{x}{f'(D)}$

$$\Rightarrow y_p = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2}\, 4\sin x - \frac{x^2}{12D+2}\, 2\cos x$$

putting $D=0$ in all

$$y_p = \frac{x^2\, 3x^2}{12(0)+2} + \frac{x^2\, 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2} + 2x^2\sin x - x^2\cos x$$

So putting in equation ①

$$y = C_1 + C_2\cos x + C_3\cos x + \tfrac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

$$\boxed{y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \tfrac{3}{2}x^4}$$

Answer.

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