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Section A

Subject Applied Calculus

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Question NO 1

ANSWER :)

Sol:

Coordinate of $P = (4, 1, 3)$ and $Q = (1, 2, 4)$;

$OP = (4i, 1j, 3k)$ & $OQ = (1i, 2j, 4k)$

$\Rightarrow \vec{OQ} - \vec{OP} = (1i, 2j, 4k) - (4i, 1j, 3k)$

$\Rightarrow \boxed{\vec{OQ} - \vec{OP} = (-3i + 1j + 1k)}$

Now distance between P & Q = $|PQ|$

$P = (4, 1, 3)$ & $Q = (1, 2, 4)$

$|PQ| = \sqrt{(4-1)^2 + (1-2)^2 + (3-4)^2}$

$|PQ| = \sqrt{(3)^2 + (1)^2 + (1)^2}$

$|PQ| = \sqrt{9 + 1 + 1}$

$\boxed{|PQ| = \sqrt{11}}$

let M be the point which divided PQ in ratio 1:3, Then by the ratio theorem position vector of M = $\overrightarrow{OM}$

$$M = \frac{3(4i + 1j + 3k) + (1)(1 + 2j + 4k)}{1+3}$$

$$M = \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$\boxed{M = \frac{13i + 5j + 13k}{4}}$$

Hence Solved the required.

Question NO 2

ANSWER

Sol

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & 4x^3+10x+4 \\ & \pm 4x^3 \\ \hline & -2x^2+10x+4 \\ & \mp 2x^2 \pm x \\ \hline & 11x+4 \end{array}$$

So $2x-1+\dfrac{11x+4}{2x^2+x} = \dfrac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+2} = \int 2x-1+\int \frac{11x+4}{2x^2+x} \longrightarrow (i)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow (ii)$$

Now find

$$= \int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$= \frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow (A)$$

$$= \frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{x\cancel{(2x+1)}}$$

$$= 11x+4 = A(2x+1)+Bx \longrightarrow (3)$$

put $x=0$ in (3)

$$\boxed{4 = A}$$

Now put $x = -1/2$ in (3)

$$11(-1/2)+4 = B(-1/2)$$

$$-\frac{11}{2}+4 = \frac{-B}{2}$$

$$\frac{-11+8}{2} = \frac{-B}{2}$$

$$-3 = -B \Rightarrow \boxed{B=3}$$

putting the value of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on both sides.

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

putting These value in eq ②

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

Now put The value in ①

$$\boxed{\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C}$$

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Question no : 3 Part (A)

ANSWER

Sol:

$$\int_0^2 x^2 e^x \, dx$$

Now first we find Integration.

$$= \int x^2 e^x \, dx$$

$$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) \, dx$$

$$= x^2 e^x - 2 \int x e^x \, dx$$

$$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x \right) dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2 x e^x + 2 e^x$$

Now put limits

$$= \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$$

$= \left(2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)\right)$

$= \left(\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2\right)$

$= \boxed{2e^2 - 2}$ ANS

Question no 3 (part B)

ANSWER

Sol:

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

let $u = \sqrt{x}$ so $du = \sqrt{x}/2x \; dx$

limit at $x = 1$ at $x = 2$, $u = \sqrt{2}$

The original equation in variable u become.

$$= \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

$$= \int_1^{\sqrt{2}} \frac{\sin u}{u} \cdot 2u\, dx = 2\int_1^{\sqrt{2}} \frac{\cancel{u}\cdot\sin u\, du}{\cancel{u}}$$

$$= 2\int_1^{\sqrt{2}} \sin u\, du$$

$= 2\left(-\cos u\Big|_{1}^{\sqrt{2}}\right)$

$= -2\left(\cos\sqrt{2} - \cos(1)\right)$

$= \boxed{2\cos(1) - 2\cos(\sqrt{2})}$ Ans

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Question NO 4

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Sol: The laplace equation in 3d is

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} \longrightarrow (A)$$

So $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$

$$\frac{du}{dx} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{du}{dx} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{d^2u}{dx^2} = -\left[x\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{d^2u}{dx^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow (i)$$

Now; $\frac{du}{dy} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$

$$\frac{du}{dy} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{d^2u}{dy^2} = -\left[y(-3/2)\left(x^2+y^2+z^2\right)^{-5/2}(2y)+\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{d^2u}{dy^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2} + \left(x^2+y^2+z^2\right)^{-3/2}\Big] \rightarrow (2)$$

$$\frac{du}{dz} = -\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}z)$$

$$\frac{du}{dz} = -z\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{d^2u}{dz^2} = 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} \rightarrow (3)$$

mixing (1) (2) & (3) in eq (A)

$$= 3x^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} + 3y\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} + 3z\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

$$\Rightarrow \left(x^2+y^2+z^2\right)^{-5/2}\left[3x^2-\left(x^2+y^2+z^2\right)+3y^2-\left(x^2+y^2+z^2\right)+3z^2\left(x^2+y^2+z^2\right)\right]$$

$$\Rightarrow \left(x^2+y^2+z^2\right)^{-5/2}\left[3x^2-\cancel{x^2}-\cancel{y^2}-z^2+3y^2-\cancel{x^2}-\cancel{y^2}-z^2+3z^2-\cancel{x^2}-\cancel{y^2}-z^2\right]$$

$$\Rightarrow \left(x^2+y^2+z^2\right)^{-5/2}(0) = 0$$

So The given $u(x,y,z)$ is Solution of laplace equation.

THE END THANK YOU