Assignment No # 03

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PDE (Partial Differential Equation) :-

An equation contains partial derivates of one or more depentent variables of two or more independant variables.

For example,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 2 \cdot \frac{\partial u}{\partial t}$$

$$\frac{\partial U}{\partial y} = - \frac{\partial v}{\partial x}$$

Application of PDE (Partial Differential Equtio)

PDE are used to model many systems in many diffenent field of science and engineering.

* Laplace Equation.
* Heat Equation.
* Wave Equation.

Laplace Equation :-

Laplace equation is used to describe the steady state distribution of heat in a body.
Also used to describe the steady state distribution of electrical charge in a body.

$$\frac{\partial^2 u(x,y,z)}{\partial x^2} + \frac{\partial^2 u(x,y,z)}{\partial y^2} + \frac{\partial^2 u(x,y,z)}{\partial z^2} = 0$$

Heat Equation :-

The function $u(x,y,z,t)$ is used to represent the temperature at time t in a physical body at a point with coordinates (x,y,z). α is the thermal diffusivity. it is sufficient to consider the case $\alpha = 1$

$$\frac{\partial u(x,y,z,t)}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

Wave Equation :- The function $u(x,y,z,t)$ is used to represent the displacement at time t of a particle whose position at rest is (x,y,z).
The constant c represents the propagation speed of the wave.

$$\frac{\partial^2 u(x,y,z,t)}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

Examples :- There is no general theory known concerning the solvability of all partial differential equations.

Such a theory is extremely unlikely to exist, given the rich variety of physical, geometric, probabilistic phenomena which can be modeled by PDE.

We will later discuss the origin and interpretation of many of these PDE.

For each $x \in U$, where U is an open subset of R^n, and $t \geq 0$: also
$DU = D_xU = (U_{x_1}, U_{x_2}, \ldots, U_{x_n})$ spatial variable $x = (x_1, \ldots x_n)$.