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Section A.

Subject Differential Equation.

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Assignment 2.

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Question No 1

The Caulty Euler equation

(1) $x^3y'' + 2x^2y' + 2y - 10x + \frac{10}{x}$

Solution:-

$$x^3 \frac{dy^3}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3y + 2x^2 D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3) + 2x^2 D + 2)y = 10 + 10x^{-1} - (i)$$

let $x = e^t \Rightarrow t = 10x$

$$xD = \Delta$$

$$x^2D^2 = D(\Delta - 1) = \Delta^2 - \Delta$$

$$x^2D^3 = \Delta(\Delta - 1)(\Delta - 2).$$

Subtituting into equation (1).

$$(\Delta - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10x + 10x$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x - 10x^6$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

Using synthetic division.

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	0

$$\Delta^2 - 2\Delta + 2 = 0$$

Now using Quadratic formula

$a = 1$, $b = 2$, $c = 2$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-2(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2(1)}$$

$$\lambda = \frac{2 \pm \sqrt{4-8}}{2(1)}$$

$$\lambda = \frac{2 \pm \sqrt{-4}}{2}$$

$$\lambda = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$\lambda = \frac{2 \pm 2i}{2}$$

$$\lambda = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$\boxed{\lambda = 1 \pm i}$$

Since roots are complex.

$$y_i = e^{x}(c_1 \cos t + c_2 \sin t).$$

Now particular integration.

$$y_p = \frac{1}{\lambda^3 - \lambda^2 + 2} \cdot 10e^t + \frac{1}{\lambda^3 - \lambda^2 + 2} \cdot 10/e^t$$

$$y_p = \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^t}{(1)^3 - (1)^2 + 2}.$$

$$y_p = \frac{\overset{5}{\cancel{10}}e^t}{\cancel{2}} + \frac{\overset{5}{\cancel{10}}e^t}{\cancel{2}}$$

$$y_p = 5e^t + 5e^t.$$

General Solution:

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + 2 \sin t) + 5e^t + 5e^t$$

Put $e^t = x$ and $t = \ln x$.

$$\boxed{y = e^{-x}(c_1 \ln x + c_2 \sin t) + 5e^t + 5e^t}$$

Question No 2.

2 $x^3 \frac{d^3y}{dx} + 4x^2 \frac{d^2y}{dx} - 5x\frac{dy}{dx} - 15y = x^4$

Solution:-

$$\text{Let } \frac{d}{dx} = D.$$

$x^3D^3y + 4x^2D^2y - 5xDy - 5xDy - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

Let $x = e^t \Rightarrow t = \ln x$

$xD = D.$

$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta.$

Now Subtituting.

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

$\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2 - \Delta) - 5(\Delta) - 15)y = e^{4t}$

$(\Delta^3 + \Delta^2 - 7\Delta - 15)y = e^{4t}.$

Synthetic Division:-

	1	+1	-7	-15
5		3	15	15
	1	4	5	0

$\Delta^2 + 4\Delta + 5 = 0.$

Quadratic Formula:-

Quadratic Formula:

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$\lambda = \frac{-4 \pm \sqrt{16-20}}{2}$$

$$\lambda = \frac{-4 \pm \sqrt{-4}}{2}$$

$$\lambda = \frac{-4 \pm 2i}{2}$$

$$\lambda = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

$$\boxed{\lambda = -2 \pm i}$$

$$y_c = e^{ut} (c_1 \cos t + c_2 \sin t).$$

For $y_p = ?$

$$y_p = \frac{1}{\lambda^3 + \lambda^2 - 7\lambda - 15} \cdot e^{ut}$$

$$y_p = \frac{1}{(4)^3 + (4)^2 - 7(4) - (15)} \cdot e^{ut}$$

$$y_p = \frac{1}{64 + 16 - 28 - 15} \cdot e^{ut}.$$

$$y_p = \frac{1}{80-43} \cdot e^{4t}.$$

$$y_p = \frac{1}{37} \cdot e^{4t}.$$

Hence,

$$y = y_c + y_p.$$

$$y = (c_1 \cos t + 2 \sin t) + \frac{1}{37} \cdot e^{4t}.$$

Again Put $= \ln x$ and $x = \ln x$.

$$\boxed{y = e^{3t} (c_1 \cos t \ln x + c_2 \sin \log x + \frac{1}{37} e^{4x}}$$

Ans.

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Question No 3.

$x^2 y'' + 2xy' - 6y = 10x^2$

Solution:-

$y(1) = 1$ and $y'(1) = -6$

$x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 10x^2$

$\Rightarrow \left(x^2\frac{d^2}{dx^2} + \frac{2xd}{dx} - 6\right)y = 10x^2$

Put $xD = \Delta \rightarrow x^2D^2 = \Delta(\Delta-1) - \Delta^2 -$

$x = e^t$ and $\log x = t$.

$(\Delta^2 - \Delta + 2\Delta - 6)y = 10e^{2t}$.

$(\Delta^2 + \Delta - 6)y = 10e^{2t}$

The characteristic equation.

$\Delta^2 + \Delta - 6 = 0$

$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$

$\Rightarrow \Delta(\Delta+3) - 2(\Delta+3) = 0$.

$\Rightarrow (\Delta+3)(\Delta-2) = 0$.

$\Delta + 3 = 0, \quad \Delta - 2 = 0$.

$\Delta = -3, \quad \Delta = 2$.

$\Delta = 2, \quad \Delta = -3$.

Since roots are real and distinct for $y_c = ?$

$y_c = c_1 e^{-3t} + c_2 e^{2t}$

For $y_p = ?$

$y_p = \frac{1}{\Delta^2 - \Delta - 6} \cdot 10e^{2t}$.

$$y_p = \frac{10}{\Delta^2-\Delta-6} e^{2t}$$

$$y_p = 10 \frac{1}{0} e^{2t}$$

Now

$$10 \frac{1}{d/d\Delta(\Delta^2-\Delta-6)} e^{2t}.$$

$$\Rightarrow 10 \frac{t}{\Delta_2+1} e^{2t}$$

$$\Rightarrow 10 \frac{1 \cdot t}{4+1} e^{2t}$$

$$\boxed{y_p = 2t e^{2t}}$$

General Solution:-

$$y = y_c + y_p.$$

$$= C_1 e^{-3t} + C_2 e^{2t} + 2te^{2t}$$

$$y = C_1 x^{-3} + C_2 x^2 + 2(\log x) x^2 \rightarrow (B).$$

Put $y(1)$ i.e $x = 1, y = 1$ in (B).

$$1 = C_1(1)^{-3} + C_2(1)^2 + 2\log(1)$$

$$1 = C_1 + C_2 \rightarrow (1)$$

Now differentiates equation (B). w.r.t x.

$$y' = -3c_1 x^{-4} + 2(2x + 2/x (x^2) + 4x \log x$$

Now put $y'(1) = -6$

i.e $y' = -6$ and $x = -6$.

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$$-6 = -3c_1 + 2c_2 + 2 + 0\}$$

$$-6 = -3c_1 + 2c_2 + 2.$$

$$-6-2 = -3c_1 + 2c_2 + 2.$$

$$\boxed{-8 = -3c_1 + 2c_2} \longrightarrow (2)$$

xing eq (1) with eq (2) and -ing from D.

$$2c_1 + \cancel{2c_2} = 2$$

$$\mp 3c_1 \pm \cancel{2c_2} = \pm 8$$

$$5c_1 = 10$$

$$c_1 = \frac{\cancel{10}\,2}{\cancel{5}\,1}$$

$$\boxed{c_1 = 2}$$

$$-8 = -3(2) + 2c_2$$

$$-8 = -6 + 2c_2$$

$$2c_2 = -8 + 6.$$

$$\boxed{2c_2 = -2}$$

$$c_2 = \frac{-\cancel{2}}{\cancel{2}}$$

$$\boxed{c_2 = -1}$$

Now Put the value of c_1 and c_2 in equation (B).

$$y = 2x^{-3} - x^2 + 2\ln x \, x\ln x^2$$

$$\boxed{y = \frac{2}{x^3} - x^2 + 2x^2 \log x}.$$

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Question 4.

$$x^2 y'' + 7xy' + 5y = x^5$$

$y(1) = 2$ and $y'(1) = 2$.

Solution:-

$$x^2 \frac{dy^2}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2 \frac{d^2}{dx} + 7x \frac{d}{dx} + 5 \right) y = x^5 \longrightarrow (A).$$

Put $xD = \Delta \Rightarrow x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$

$x = e^t \Rightarrow \log x = t$ in eq (2)

$$\Rightarrow (\Delta^2 - \Delta + 7\Delta + 5) y = e^{5t}$$

$$\Rightarrow (\Delta^2 + 6\Delta + 5) y = e^{5t}$$

By Quadratic Formula.

$$\Delta = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\Delta = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}.$$

$$\Delta = \frac{-6 \pm \sqrt{36 - 20}}{2}.$$

$$\Delta = \frac{-6 \pm \sqrt{16}}{2}.$$

$$\Delta = \frac{-6 \pm \sqrt{4^2}}{2}.$$

$$\Delta = \frac{-\cancel{2} \pm (-3 + 2)}{\cancel{2}}$$

$$\boxed{\Delta = -3 \pm 2}$$

Since roots are real and distinct:

$$y_c = C_1 e^{5t} + C_2 e^{t}$$

For $y_p = ?$

$$y_p = \frac{1}{\Delta^2 + 6\Delta + 5} \cdot e^{5t}$$

$$y_p = \frac{1}{(5)^2 + 6(5) + 5} \cdot e^{5t}$$

$$y_p = \frac{1}{60} \cdot e^{5t}.$$

Now general solution is

$$y = y_c + y_p$$

$$y = C_1 e^{-5t} + C_2 e^{t} + \frac{1}{60} e^{5t}$$

$$y = C_1 x^{-5} x^{-1} + \frac{1}{60} x^5 \longrightarrow (A)$$

$x = 0$ put in this equation.

No in equation (B) $e^0 = 1$.

Put $y(0) = 2$ i.e $y = 2$ and $x = 2$

$$2 = C_1 (2)^{-5} + (2(2)^{-1} + \frac{1}{60}(2)^5$$

$$2 = -32 C_1 = 2(2 + \frac{1}{\cancel{60}_{15}}(\cancel{32}^{8})$$

$$2 = -32 C_1 - 2 C_2 + \frac{8}{15}$$

$$2 - \frac{8}{15} = -32 C_1 - 2 C_2$$

$$\frac{22}{15} = -32 C_1 - 2 C_2 \longrightarrow (C)$$

Now differentiate eq (B) w.r.t (x).

$$y' = -5C_1 x^{-6} - C_2 x^{-2} + \frac{1}{12} x^4 \rightarrow$$

Put $y'(1) = 2$ i.e $y' = 2$ and $x = 2$.

in above equation

$2 = -5c_1\, c_2^{-6} - c_1(2)^{-2} + 1/12\,(2)^4$

$2 = -5c_1(-64) - c_2(4) + 1/12\, c_1 6.$

$2 = 320c_1 + 4c_2 + 4/3.$

$\Rightarrow 2 - 4/3 = 320c_1 + 4c_2$

$\Rightarrow 2/3 = 320c_1 + 4c_2 \longrightarrow D.$

xing eq C with 2 and then -ing eq C from D.

$-\frac{44}{15} = 64c_1 + 4c_2$

$-44/15 = 64c_1 + 4\cancel{c_2}$

$\pm 2/3 = +320c_1 \pm \cancel{4}c_2$

$34/15 = -256\,c_1.$

$c_1 = \frac{34}{15} \times 256.$

$\boxed{c_1 = 580}$

Put the value of c_1 in eq C.

$22/15 = -32(520) - 2c_2.$

$\Rightarrow 22/15 = -18560 = 2c_2.$

$\Rightarrow 22/15 + 18560 = 2c_2$

$\Rightarrow \frac{18561}{2} = c_2.$

$\Rightarrow \boxed{-9280 = c_2}$

Put the value of c_1 & c_2 in eq B.

$y = 580x^{-5} - 9280x^{-1} + 1/60\, x^5$

$$\boxed{y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5}$$

Question no 5.

$(x+1)^2 y'' - 3(x+1)^1 y' + 4y = x^2$

Solution:

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow \left((x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4\right)y = x^2$$

$$((x+1)^2 \Delta^2 - 3(x+1)\Delta + 4)y = x^2 \longrightarrow (A)$$

Put $(x+1)\Delta = \Delta \Rightarrow (x+1)^2 \Delta^2 = \Delta$

$\Delta(\Delta - 1)\ \Delta^2 - \Delta.$

$x = e^t$ in eq (A)

$$\Rightarrow (\Delta^2 - \Delta - 3\Delta + 4)y = e^{2t}$$

$$\Rightarrow (\Delta^2 - 4\Delta + 4)\quad y = e^{2t}$$

$$\Rightarrow (\Delta^2 - 4\Delta + 4)\quad 2 = e^{2t}$$

For y_c we find the roots.

$$\Delta^2 - 4\Delta + 4 = 0$$

$$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$$

$$\Delta(\Delta - 2) - 2(\Delta - 2) = 0$$

$$\Delta - 2 = 0, \quad \Delta - 2.$$

$D - 2 = 0$, $D = 2$.

So the roots are real and repeat

the general solution are.

$$y = (C_1 + C_2 x)^{2x}$$

$$y = (C_1 + C_2 x)^{2x}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 - 4D + 4}$$

$(2)^2 - 4(2) + 4$

$\Rightarrow 0$.

$$y_p = \frac{2}{2D - 4} e^{2t}$$

If we put "2"

$$2D - 4 \Rightarrow 2(2) - 4 = 0$$

we take again derivation.

$$y_p = \frac{\cancel{2}}{\cancel{2}} e^{2t}$$

General solution

$$y_p = C_1 + C_2 x)^{2t} + e^{2t}$$

Ans.