FINAL TERM EXAMINATION

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SUBJECT ⇒ NUMERICAL ANALYSIS

SUBMITTED TO ⇒ MAM SHUMAILA

DEPARTMENT ⇒ CIVIL ENGINEERING

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Question :- 01

$$\frac{dy}{dx} = 2x \quad ; \quad y(0) = 1$$

$$h = 0.1$$

Solution :-

$$f(x,y) = 2x$$

$$x_0 = 0 \quad , \quad y_0 = 1$$

$$h = 0.1$$

$$x_{n+1} = x_n + h$$

put $n = 0$

$$x_1 = x_0 + h$$
$$= 0 + 0.1$$
$$= 0.1$$
$$x_2 = x_1 + h$$
$$= 0.1 + 0.1$$
$$= 0.2$$
$$x_3 = 0.3$$
$$x_4 = 0.4$$
$$x_5 = 0.5$$

1st Iteration :-

Euler's formula

$$y_{n+1}^{*} = y_n + hf(x_n, y_n)$$

$n = 0$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1[(0)(1)]$$

$$= 1.1$$

Modified Euler's formula

$$y_{n+1} = y_n + h/2 \left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^{*}) \right]$$

$$y_1 = y_0 + h/2 \left[f(x_0, y_0) + (x_1, y_1^{*}) \right]$$

$$y_1 = 1 + \frac{0.1}{2} \left[(0)(1) + (0.1)(1.1) \right]$$

$$y_1 = 1 + 0.05 \left[0 + 1.1 \right]$$

$$= 1 + 0.05 \left(\frac{2.2}{2} \right)$$

$$= 1.11$$

2nd Iteration :-

Euler's formula

$$y_2^* = y_1 + hf(x_1, y_1)$$

$$y_2^* = 1.11 + 0.1[(0.1)(1.11)]$$

$$y_2^* = 1.11 + 0.1(1.21)$$

$$= 1.11 + 0.121$$

$$= 1.231$$

Modified Euler's formula

$$y_2 = y_1 + 0.1/2\,[x_1, y_1] + [x_2, y_2^*]$$

$$= 1.11 + 0.05[(0.1)(1.11) + (0.2)(1.231)]$$

$$= 1.11 + 0.05[1.21 + 1.431]$$

$$= 1.11 + 0.05(2.641)$$

$$= 1.242$$

3rd Iteration :-

$$y_3^* = y_2 + hf(x_2, y_2)$$

$$y_3^* = 1.202 + 0.1(0.2)(1.242)$$

$$= 1.242 + 0.1442$$

$$= 1.3862$$

Modified Euler's Method

$$y_3 = y_2 + 0.1/2 \left[(x_2, y_2) + (x_3, y_3^*)\right]$$

$$= 1.242 + 0.05 \left[(0.2) + (1.242) + (0.3) + (1.3862)\right]$$

$$= 1.242 + 0.05 \left[1.442 + 1.6862\right]$$

$$= 1.398$$

4th Iteration :-

$$y_4^* = y_3 + hf(x_3, y_3)$$

$$y_4^* = 1.398 + 0.05(0.3 + 1.398)$$

$$= 1.483$$

Modified Euler's Method

$$y_4 = y_3 + 0.05 \left[(x_3, y_3) + (x_4, y_4^*)\right]$$

$= 1.398 + 0.05 [0.3 + 1.398] + [0.4 + 1.483]$

$= 1.398 + 0.05 (1.698 + 1.883)$

$= 1.577$

5th Iteration :-

$y_5^* = y_4 + hf(x_4, y_4)$

$= 1.577 + 0.05(0.4 + 1.577)$

$= 1.675$

Modified Euler's Method.

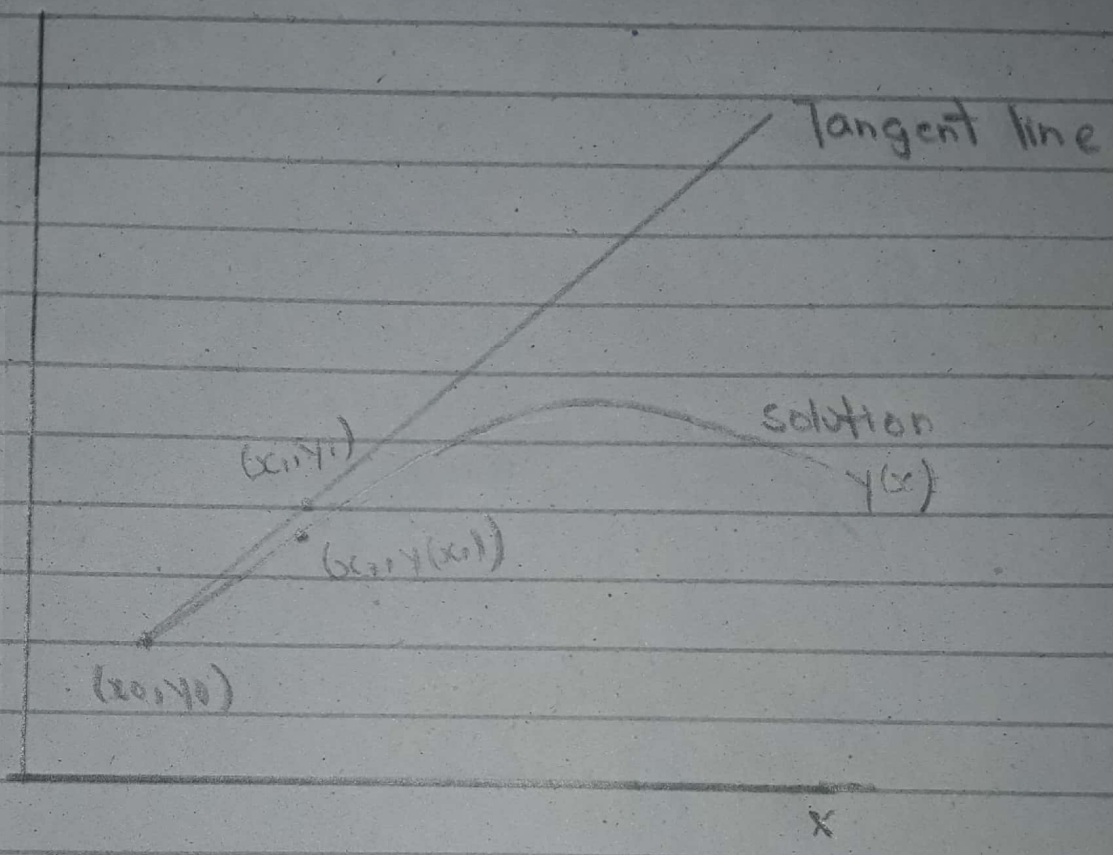
$y_5 = y_4 + 0.05 [x_4, y_4] + [x_5, y_5^*]$

$= 1.577 + 0.05 [0.4 + 1.577] + [0.5 + 1.675]$

$= 1.577 + 0.05 [4.152]$

$= 1.785$

5(a)



We use Euler's method to approximate solutions to a couple of first order differential equation, with a step size of $h = 0.1$

$x = 0.1, 0.2, 0.3, 0.4, 0.5$, By those points we can compare values of solutions at these points.

Question :- 02

(02.) Use Ruge kutta Method ...

$$\frac{dy}{dx} = x^2 + x - y$$

Subject to $y = 0$ when $x = 0$, for $0 \leq x \leq 0.6$

Given Data :-

$y = 0$, $x = 0$, $h = 0.2$ $\quad 0 \leq x \leq 0.6$

$$y_{n+1} = y_n + k$$

1st Iteration :-

put $n = 0$

$$y_1 = y_0 + k \quad , k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_1 = hf(x_0, y_0)$$

$$k_1 = 0.2\,(0^2 - 0 - 0)$$

$$\boxed{k_1 = 0}$$

$$k_2 = hf(x_n + h/2, y_n + h/2)$$

$$= 0.2\, f(x_0 + h/2, y_0 + h/2)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$$

$= 0.2\, f(0.1, 0.1)$

$= 0.2(0.1^2 + 0.1 - 0.1)$

$\boxed{k_2 = 0.0020}$

$k_3 = hf\ (x_n + h/2,\ y_n + k_2/2)$

$= 0.2f\left(0 + 0.2/2,\ 0 + \frac{0.002}{2}\right)$

$= 0.2f\ (0.1,\ 0.001)$

$= 0.2\ (0.1^2 + 0.1 - 0.001)$

$\boxed{k_3 = 0.0218}$

$k_4 = hf\ (x_n + h,\ y_n + k_3)$

$= 0.2f\ (0 + 0.2,\ 0 + 0.0218)$

$= 0.2f\ (0.2, 0.0218)$

$= 0.2\ (0.2^2 + 0.2 - 0.0218)$

$\boxed{k_4 = 0.0436}$

$k = \frac{1}{6}\ (0 + 2(0.002) + 2\ (0.0218) + 0.0436)$

$k = 0.0152$

$y_1 = 0 + 0.0152$

$y_1 = 0.0152$

Question :- 03

Trapezoidal Rule

Given Data :-

$a = 0$, $b = 10$, $n = 10$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution :-

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

Using formula

$$\int f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \cdots f(x_9) + f(x_{10})\right]$$

$$= \frac{1}{2}\{10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 62.1 + 60.3 + 61.2\}$$

$$= \boxed{412.9 \text{ Answer}}$$

Question :- 04

$$\int_2^3 \ln(x^3+1)\,dx$$

use 10 strips By SIMPSON'S RULE

Solution :-

$$n = 10$$

$$h = \frac{b-a}{n}$$

$$h = \frac{3-2}{10}$$

$$h = 0.1$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0)+4(f(x_1)+f(x_3)\cdots\right] + 2\left[f(x_2\cdots)+f(x_n)\right]$$

$$= \frac{0.1}{3}\left[0.693+4(0.846+1.162+1.476+1.777)+2(1.003+1.320+1.628+1.922)+2.062\right]$$

$$= 1.184 \text{ Answer}$$