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Question NO # 1 :→

Answer :→ By Euler's Method:.

Given Data :→

$$y(0) = 1 \;, h = 0.1, \; x_0 = 0$$

→ By formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h[2x_n]$$

1st Iteration :→

$$n = 0$$

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1(2(0))$$

$$y_1 = 1 + 0.1(0)$$

$$y_1 = 1 + 0$$

$$\boxed{y_1 = 1}$$

$$x_{n+1} = x_n + h$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

⇒ (P-T-O)

2nd Iteration :→

$$n = 1$$

$$y_2 = y_1 + h(2x_1)$$

$$y_2 = 1 + 0.1(2(0.1))$$

$$y_2 = 1 + 0.1(0.2)$$

$$y_2 = 1 + 0.02$$

$$\boxed{y_2 = 1.02}$$

$$x_{n+1} = x_n + h$$

$$x_2 = x_1 + h$$

$$x_2 = 0.1 + 0.1$$

$$\boxed{x_2 = 0.2}$$

3rd Iteration :→

Put $n = 2$

$$y_3 = y_2 + h(2x_2)$$

$$y_3 = 1.02 + 0.1(2(0.2))$$

$$y_3 = 1.02 + 0.1(0.4)$$

$$y_3 = 1.02 + 0.04$$

$$\boxed{y_3 = 1.06}$$

(P-T-O)⇒

$$x_{n+1} = x_n + h$$

Put $n = 2$

$$x_{2+1} = x_2 + h$$

$$x_3 = x_2 + h$$

$$x_3 = 0.2 + 0.1$$

$$\boxed{x_3 = 0.3}$$

(B) By Modified Euler method.

$$dy/dx = 2x$$

Given data :→

$$y_0 = 1, \; x_0 = 0, \; h = 0.1$$

Formula

$$y^*_{n+1} = y_n + h\left[f(x_n)\right]$$

$$y^*_{n+1} = y_n + h(2x_n) \longrightarrow (1)$$

$$y_{n+1} = y_n + h/2\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

$$= y_n + h/2\left[2x_n + 2x_n\right]$$

$$= y_n + h/2\left[4x_n\right]$$

1st Itration :→

$n = 0$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$y_1 = y_0 + h/2\,(4x_0)$

$y_1 = 1 + \frac{0.1}{2}(4(0))$

$y_1 = 1 + \frac{0.1}{2}(0)$

$y_1 = 1 + 0$

$\boxed{y_1 = 1}$

2nd Itration :→

$$n = 1$$

$$x_2 = x_1 + h$$

$$x_2 = 0.1 + 0.1$$

$$\boxed{x_2 = 0.2}$$

$$y_2 = y_1 + h/2\,(4x_1)$$

$$y_2 = 1 + \frac{0.1}{2}(4(0.1))$$

$$y_2 = 1 + \frac{0.1}{2}(0.4)$$

$$\boxed{y_2 = 1.02}$$

3rd Itration :→

$$n = 2$$

$$x_3 = x_2 + h$$

$$x_3 = 0.2 + 0.1$$

$$x_3 = 0.2 + 0.1$$

$$\boxed{x_3 = 0.3}$$

$$y_3 = y_2 + h/2\,(4x_2)$$

$$= 1.02 + \frac{0.1}{2}(4(0.2))$$

$$\boxed{y_3 = 1.06}$$

Question #2 :→

Answer :→

Given Data :→

$$y=0,\ x=0,\ h=0.2,\ 0\le x\le 0.6$$

$$y_{n+1} = y_n + k$$

1st Iteration :→

$$n = 0$$

$$y_1 = y_0 + k\ ,\quad k = \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_1 = h(x_0^2 - x_0 - y_0)$$

$$k_1 = 0.2(0^2 - 0 - 0)$$

$$\boxed{k_1 = 0}$$

$$k_2 = hf\left(x_n + h/2,\ y_n + h/2\right)$$

$$= 0.2f(x_0 + h/2,\ y_0 + h/2)$$

$$= 0.2f\left(0 + \frac{0.2}{2},\ 0 + \frac{0.2}{2}\right)$$

$$= 0.2f(0.1,\ 0.1)$$

$$= 0.2\left(0.1^2 + 0.1 - 0.1\right)$$

$$\boxed{k_2 = 0.0020}$$

(P-T-O)⇒

$$K_3 = hf\left(x_n + h/2 , y_n + \frac{K_2}{2}\right)$$

$$= 0.2f\left(0 + \frac{0.2}{2} , 0 + \frac{0.002}{2}\right)$$

$$= 0.2f(0.1 , 0.001)$$

$$= 0.2(0.1^2 + 0.1 - 0.001)$$

$$\boxed{K_3 = 0.0218}$$

$$K_4 = hf(x_n + h , y_n + K_3)$$

$$= 0.2f(0 + 0.2, 0 + 0.0218)$$

$$= 0.2f(0.2 , 0.0218)$$

$$= 0.2(0.2^2 + 0.2 - 0.0218)$$

$$\boxed{K_4 = 0.0436}$$

$$K = 1/6\,(0 + 2(0.002) + 2(0.0218) + 0.0436)$$

$$K = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$\boxed{y_1 = 0.0152}$$

Q No #3 :→

Given Data :→

$$a = 0,\ b = 10,\ n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution :→

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

using formula

$$\int f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\left(f(x_1)+f(x_2)+f(x_3)\cdots f(x_9)+f(x_{10})\right]\right.$$

$$= \frac{1}{2}\left[10.1 + 2(17.2+24.4+29.2+34.6+41.2+50.9+57.8+60.3+61.2) + 62.1\right]$$

$$= 412.9$$

Answer :→ $= 412.9$

Q No # 4 :→ $\int_2^3 \ln(x^3+1)\,dx$

Answer :→ use 10 strips.

$a = 2, \quad b = 3$

$f(x) = \ln(x^3+1)$ Strips = 10

$$\Delta x = \frac{b-a}{2n}$$

$$\Delta x = \frac{3-2}{2(5)} = \frac{1}{10} = 0.1$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
x	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
$f(x)$	2.19	2.32	2.45	2.57	2.69	2.81	2.92	3.02	3.13	3.23	3.33

$$A \approx \frac{\Delta x}{3}\int f(x_0) - 2\Big[f(x_2 + f(x_4) + f(x_6) + f(x_8) + 4\{f(x_1) + f(x_3) + f(x_5) + f(x_7) + f(x_9)\Big] + f(x_{10})\Big]$$

$$A = \frac{0.1}{3}\Big[2.19 + 2\big[2.45 + 2.69 + 2.69 + 2.92 + 3.13\big] + 4\big[2.32 + 2.57 + 2.81 + 3.02 + 3.23\big] + [3.33]\Big]$$

$$A = 0.3\big[2.19 + 22.38 + 55.8 + 3.33\big]$$

$$A \approx 2.511$$

$$\boxed{\int_2^3 \ln(x^3+1\,dx = 2.511} \leftarrow \text{Ans}$$