Mid term Exam
Summer

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Section:- B

Dept:- BE(CE)

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Q.1

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \leq 1 \leq 3 \\ 2t + 3 & 3 < 1 \leq 4 \\ 12 & t > 4 \end{cases}$$

(a) State any point of discontinuity

(b) Find if they exist

i) $\lim\limits_{t \to 3} g$

Sol:

(2)

(a) To check possibility of the discontinuity of the function is at $t = 0$ & 4

→ First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$

$= \lim_{h \to 0} 1 + h^2 + 2h$

③

Apply limit

$= 1+0^2+2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t+3$

$= \lim_{h \to 0} 2(1-h)+3$

$= \lim_{h \to 0} 2-2h+3$

Apply limit

$= 2-2(0)+3$

$= 5$

③ ④

$R.H.L \neq L.H.L = g(t) = 5$

$\Rightarrow$ Now at $t = 4$

$$g(4) = 2(4) + 3$$
$$= 8 + 3$$
$$= 11$$

For R.H.L

$$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$$

$$= \lim_{h \to 0} 2 + 2h + 3$$

Apply limit

$$= 2 + 2(0) + 3 \Rightarrow 5$$

④ ⑤

For L.h.L

$$\lim_{h \to 0} g(1-h) = 12$$

$g(4) = $ R.H.L $\neq$ L.H.L

point of discontinuity is at $t=4$

(b) Find, if they exist

(i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L

$$\lim_{h \to 3} (1+h) = \lim_{h \to 3} (1+h)^2$$

(6)

$$= \lim_{h \to 3} 1 + h^2 + 2h$$

Apply limits

$$= 1 + 3^2 + 2(3) \Rightarrow 16$$

L.H.L

$$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t + 3$$

$$= \lim_{h \to 3} 2(1-h) + 3$$

$$= \lim_{h \to 3} 2 - 2h + 3$$

Apply limit

$$2 - 2(3) + 3$$

$$= 2 - 6 + 3$$

$$= -1$$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is -ve)

Q.No (2) ⑦

(i) Find the Maclarin's Series for

$$y(x) = x^2 + \sin x$$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

let

$y(x) = x^2 + \sin x$

$y'(x) = 2x + \cos x$

$y''(x) = 2 - \sin x$

$y'''(x) = -\cos x$

Put $x=0$ in all function

$y(0) = 0$

$y'(0) = 1$

$y''(0) = 2$

$y'''(0) = -1$

putting value in formula

$$= 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1)$$

$$= 0 + x + x^2 - \frac{x^3}{6}$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} \text{ Ans}$$

Q. NO (02) (9)

1) $1+xy=x^2+y^2$

Sal:

$$1+xy = x^2+y^2 \quad \text{—} (1)$$

diff eq (1) b/s w.r.t "x"

$$\frac{d}{dx}(1+xy) = \frac{d}{dx}(x^2+y^2)$$

$$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$= 0 + \left(x \cdot \frac{d}{dx}y + y \cdot \frac{d}{dx}x\right) = 2x + 2y \cdot \frac{dy}{dx}$$

$$= x \cdot \frac{d}{dx} + y(1) = 2x + 2y \cdot \frac{dy}{dx}$$

(10)

$= x \cdot y' + y = 2x + 2y \cdot y'$

$= x \cdot y' + y = 2x + 2y \cdot y$

$= x \cdot y' - 2y \cdot y' = 2x - y$

$= y'(x - 2y) = (2x - y)$

$$= y' = \frac{2x - y}{x - 2y}$$

Differ again w.r.t "x"

$$y'' = \frac{d}{dx}\left(\frac{2x - y}{x - 2y}\right)$$

$$= \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

$$= \frac{(x-2y)(2)\left(-\frac{dy}{dx}\right) - (2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= \frac{(2x-4y)(-y') - \left(2x-4x\frac{dy}{dx} - y + 2yy'\right)}{(x-2y)^2}$$

$$y''(x-2y)^2 = 2xy' + 2yy' - 2x + y$$

$$y''(x-2y)^2 = \left(\frac{2x-y}{x-2y}\right)(2x-2y) - 2x + y$$

$$y'' = \frac{\left(\frac{2x-y}{x-2y}\right)(2x+2y) - 2x + y}{(x-2y)^2}$$

Ans

Q.No 3

2) Find y' by using logarithmic differentiation

$$y = x^3(1+x)^9 e^{6x}$$

Sol:

Find y' by using logrithim diff

$$y = x^3(1+x)^9 e^{6x}$$

Taking $\ln$ to b/s

$$\ln y = \ln\left(x^3(1+x)^9 e^{6x}\right)$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$\ln y = 3\ln x + 9\ln(1+x) + 6x \ln e$$

$'$

(13)

Diff w.r.t x

$$\frac{d}{dx}\ln y = \frac{d}{dx}3\ln x + \frac{d}{dx}9\ln(1+x) + \frac{d}{dx}6x\ln e$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} + \frac{1}{e^{6x}}\cdot 6$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$$

$$\frac{dy}{dx} = x^3(1+x)^9 e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + \frac{6}{e^{6x}}\right)$$