IQRA NATIONAL UNIVERSITY PESHAWAR

DEPARTMENT OF CIVIL ENGINEERING

SUBJECT: APPLIED CALCULAS

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Q.NO (01) ANSWER

①

Q No: 1

The function g(t) is defined by

$g(t) = 0 \quad t < 0$

$t^2 \quad 0 \leq t \leq 3$

$2t + 3 \quad 3 < t \leq 4$

$12 \quad t > 4$

a) State any point of discontinuity

b) find if they exist.

i) $\lim_{t \to 3} g$

Sol: a) To chek possibility of discontinuity of the function is $t = 0 \,\&\, 4$

⟹ First at $t = 0 \Rightarrow g(t) = t^2 \Rightarrow g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim (1+h)^2 \Rightarrow \lim_{h \to 0} 1 + h^2 + 2h$

⇒ Apply limit

$= 1 + 0^2 + 2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t + 3$

$\lim_{h \to 0} 2(1-h) + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

Now at $t = 4$

$g(4) = 2(4) + 3$

$= 11$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} 2(1+h) + 3$

$\lim_{h \to 0} 2 + 2h + 3$

$\Rightarrow$ Apply limits

$2 + 2(0) + 3 \Rightarrow 5$

For L.H.L

$\text{Lim } g(1-h) = 12$

$g(4) = \text{R.H.L} \neq \text{L.H.L}$

Point of discontinivity is at $t = 4$

Part :B

Find if they exist

(i) $\lim_{t \to 3} g$

For $g(t) = t^3$

R.H.L

$\lim g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1+h^2+2h$

Apply limits

$1+3^2+2(3) \Rightarrow 16$

L.H.L

$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t+3$

$= \lim_{h \to 3} 2(1-h)+3$

$= \lim_{h \to 3} 2 \quad 2h+3$

$\Rightarrow$ Apply limit

$= 2-2(3)+3$

$= 2-6+3$

$= -1$

R.H.L $\neq$ L.H.L do not exist since L.H.L is -ve

--------------------END OF Q.NO 01----------------------------

Q.NO (03 Part A) **ANSWER**

Q. No (03) (i)

$1 + xy = x^2 + y^2$ to find y''

Diff w.r.t x

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2) \quad 0 + x\frac{dy}{dx} + y\cancel{\frac{dx}{dx}}$$

$$= 2x\cancel{\frac{dy}{dx}} + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{\cancel{(x - 2y)}}{\cancel{(x - 2y)}}\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

$y' = \frac{2x - y}{x - 2y}$ again dif w.r.t x

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(x-2y)}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{\cancel{2x} - xy' - 4y^2 - \cancel{2x} + 4xy' + y - 2yy'}{(x-2y)^2}$$

$$y'' = \frac{3xy' - 4y^2 - 2yy'}{(x-2y)^2}$$

--------------------END OF Q.NO 03 A-------------------------

Q.NO (03 Part B) **ANSWER**

Q NO 03 B

$$y = x^3 (1+x)^9 e^{6x}$$

$$\log y = \log (x^3 (1+x)^3 e^{6x}$$

$$\log y = \log x^3 + \log (1+x) + \log e^{6x}$$

$\log$

$$\frac{1}{y} \frac{1}{(y^2)} \frac{dy}{dx} = \frac{3x^2}{x^3} + \frac{9\cancel{(1+x)^8}}{\cancel{(1+x)^9}} + 6$$

$$\frac{1}{y^3} \frac{dy}{dx} = \frac{3}{x^2} + 9 + 6x$$

$$\frac{1}{y^3} \frac{dy}{dx} = 3 + 9x^2 + 6x^3$$

$$\frac{1}{y^3} \frac{dy}{dx} = 6x^3 + 9x^2 + 3$$

$$\frac{dy}{dx} = y^3 (6x^3 + 9x^2 + 3$$

--------------------END OF Q.NO 03 B--------------------------

Q.NO (02) **ANSWER**

(1)

Q.no 2

$Y(x) = x^2 + \sin x$

Since we know that the maclurin series is

$$Y(x) = Y(x) + Y'(x)\,(x - x_0) + \frac{Y''(x_0)(x-x_0)^2}{2!} +$$

Put $x_0 = 0$

$$Y(x) = Y(0) + (x-0)\,Y(0) + \frac{(x-0)^2 Y''(0)}{2!} +$$

$$Y(x) = Y(0) + xy'(0) + \frac{x^2 Y''(0)}{2!} + \quad (1)$$

Now find

$Y(0) = ?$

$Y(x) = x^2 + \sin x$

$Y(0) = 0 + \sin 0$

$= 0 + 0$

$Y(0) = 0$

$y(x) = x^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$

$y'(x) = 2x + \cos 0$

$0 + 1$

$y'(0) = 1$

Since $y'(x) = 2x + \cos x$

$\frac{d}{dx} y'(x) = \frac{2d}{dx} x + \frac{d}{dx} \cos x$

$= 2 - \sin x$

$y''(x) = 2 - \sin x$

$y''(0) = 2 - \sin 0$

$= 2 - 0 = 2$

$y''(0) = 2$

Now

$$y''(x) = 2 - \sin x$$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x$$

$$y'''(0) = -\cos x$$

$$y''(0) = -1$$

Put in eq (i)

$$y(x) = 0 + x(1) + \frac{x^2 (2)}{2!} + \frac{x^3 (-1)}{3!}$$

$$= x + \frac{\cancel{2} x^2}{\cancel{2}!} - \frac{x^3}{3!} + \cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \cdots$$

So

$$y(x) = x + x^2 - \frac{x^3}{3!} + \cdots$$

--------------------END OF Q.NO 02----------------------------