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Section "A"
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Q 1) The function $g(t)$ is defined by

$g(t) = 0 \quad t < 0$

$t^2 \quad 0 \leq t \leq 3$

$2t+3 \quad 3 < t \leq 4$

$12 \quad t > 4$

(a) State any point of discontinuity.

Solution:-

At point $t=3$ $g(t)$ is not contineous.

because $g(3) = 9$

and $\lim_{t \to 3^+} g(t) \neq \lim_{t \to 3^-} g(t)$

so $\lim_{t \to 3}$ doesnot exist

In this case

Thus $t=0$ is disconcontinuity point in the domain $g(t)$.

Q1) b) Find, if they exist

$\lim_{t \to 3} g$

Solution:- $g(0) = 0$ and

$\lim_{t \to 0} g(t) = 0$

and $\lim_{t \to 0} g(t) = 0$

i.e $\lim_{t \to 0} g(t) = \lim_{t \to 0} g(t)$

$\Rightarrow \lim_{t \to 0} g(t)$ exist

and $\lim_{t \to 0} g(t) = g(0)$

so $g(t)$ is contineous at point $t=0$.

Q 2) Find the Maclaurin's series for

$y(x) = x^2 + \sin x$

$f(x) = x^2 + \sin x$

$f(0) = 0$

$f'(x) = 2x + \cos x$

$f'(0) = 2(0) + \cos(0)$

$f'(0) = 1$

$f''(x) = 2 - \sin x$

$f''(0) = 2 - \sin(0)$

$f''(0) = 2$

$f'''(x) = -\cos x$

$f'''(0) = -\cos(0)$

$f'''(0) = -1$

Substituting these values in the formula.

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)\, x^3}{3!} + \cdots$$

$$x^2 + \sin x = 0 + 1x + \frac{\cancel{2}x^2}{\cancel{2!}} + \frac{((-1)x^3)}{3!} + \cdots$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \cdots$$

Q3) Find y'' given

$$1 + xy = x^2 + y^2$$

$$\frac{d}{dx}(1) + \frac{d}{dx} xy = \frac{d}{dx} x^2 + \frac{d}{dx} y^2$$

$$0 + x \cdot \frac{dy}{dx} + y = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x - 2y) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

Now taking second derivative using quotients formula.

$$y'' = \frac{v \cdot u' - u \cdot v'}{v^2}$$

$u = 2x - y$

$u' = 2$

$v = x - 2y$

$v' = 1$

put in formula.

$$y'' = \frac{(x-2y)(2) - (2x-y)(1)}{(x-2y)^2}$$

$$y'' = \frac{2x - 4y - 2x + y}{(x-2y)^2}$$

$$y'' = \frac{-3y}{(x-2y)^2}$$

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(Q3) ii) Find y' by using logarithmic differentiation. Page 5

$$y = x^3 (1+x)^9 e^{6x}.$$

Taking log on both side.

$$\log y = \log x^3 (1+x)^9 e^{6x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^3(1+x)^9 e^{6x}} \cdot \frac{d}{dx}\left(x^3(1+x)^9 e^{6x}\right)$$

$$\frac{dy}{dx} = y' = \frac{y}{x^3(1+x)^9 e^{6x}} \left[x^3(1+x)^9 \frac{d}{dx} e^{6x} + x^3 \cdot e^{6x} \frac{d}{dx}(1+x)^9 + (1+x)^9 e^{6x} \frac{d}{dx}(x^3)\right]$$

$$= \cancel{x^3 (1+x}$$

$$= \frac{y}{x^3(1+x)^9 e^{6x}} \left[x^3(1+x)^9(6e^{6x}) + 9x^3 e^{6x}(1+x)^8 + 3x^2 e^{6x}(1+x)^9\right]$$

$$= \frac{y}{x^3(1+x)^9 e^{6x}} \left[6x^3 e^{6x}(1+x)^9 + 9x^3 \cdot e^{6x}(1+x)^8 + 3x^2 e^{6x}(1+x)^9\right]$$

$$= \frac{y}{x^{\cancel{3}}(1+x)^{\cancel{9}}\cancel{e^{6x}}} \left(3\cancel{x^2}\cancel{e^{6x}}\cancel{(1+x)^8}(2x(1+x)^9 + 3x + (1+x)\right)$$

$$= \frac{y}{x(1+x)} \left(3\left(2x(1+x)^9 + 4x + 1\right)\right)$$

$$= \frac{3y\left[(2x(1+x)^9 + 4x + 1)\right]}{x(1+x)}$$

Q3 ii) Find y' by using logarithmic differentiation.

$$y = x^3(1+x)^9 e^{6x}$$

$$y' = x^3(1+x)^9 \frac{d}{dx} e^{6x} + x^3\left(\frac{d}{dx}(1+x)^9\right) \cdot e^{6x}$$
$$+ \left(\frac{d}{dx} x^3\right)\left((1+x)^9\right) \cdot e^{6x}.$$

$$= x^3(1+x)^9(6e^{6x}) + x^3(9(1+x)^8) \cdot e^{6x}$$
$$+ 3x^2(1+x)^9 \cdot e^{6x}.$$

$$= 6x^3e^{6x}(1+x)^9 + 9x^3 \cdot e^{6x}(1+x)^8 + 3x^2e^{6x}(1+x)^9$$

$$= 3x^2e^{6x}(1+x)^8\left[2x(1+x)^9 + 3x + (1+x)\right]$$

$$= 3x^2e^{6x}(1+x)^8\left[2x(1+x)^9 + 3x + 1 + x\right]$$

$$= 3x^2e^{6x}(1+x)^8\left[2x(1+x)^9 + 4x + 1\right].$$