

Course Title E-N-A

Module 4th

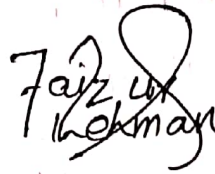
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Q1

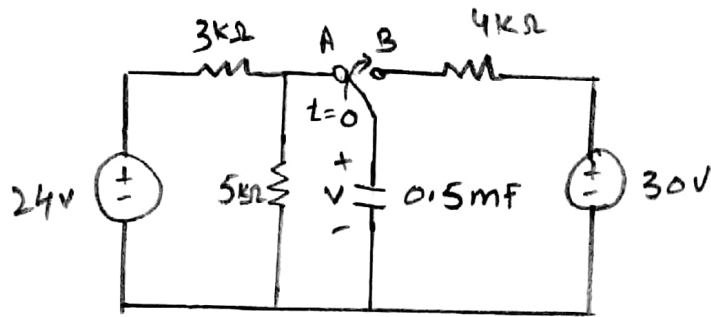


Figure 1

Solution:- For $t < 0$ the switch is at position A. Capacitor act like an open circuit to dc. but V is same as the voltage across $5k\Omega$ resistor. Hence the voltage across the capacitor just before $t=0$ is obtained by voltage division as.

$$V(0^-) = \frac{5}{5+3} (24)$$

$$V(0^-) = \frac{5}{8} (24)$$

$$V(0^-) = 15 \text{ V}$$

Using the fact that capacitor voltage can't change instantaneously.

$$V(0^-) = V(0) = V(0^+) = 15 \text{ V}$$

$t > 0$ the switch is position B. Thevenin resistance connect to the capacitor $R_{Th} = 4k\Omega$ the time const is

$$\tau = R_{Th} C = 4 \times 10^3 \times 0.5 \times 10^{-3}$$

$$\tau = 2 \text{ s}$$

Since the capacitor act like an open circuit to dc at steady state

$$V(\infty) = 30 \text{ V}$$

Thus

$$\begin{aligned}
 V(t) &= v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \\
 &= 30 + (15 - 30)e^{-t/2} \\
 &= 30 + (-15)e^{-t/2} \\
 &= (30 - 15e^{-0.5t}) \text{ V}
 \end{aligned}$$

$$\text{At } t = 2, \quad V(2) = 30 - 15e^{-\frac{2}{2}}$$

$$V(2) = 30 - 15e^{-1}$$

$$V(2) = 24.48 \text{ V}$$

$$\text{At } t = 8, \quad V(8) = 30 - 15e^{-\frac{8}{2}}$$

$$V(8) = 30 - 15e^{-4}$$

$$V(8) = 29.72 \text{ V}$$

Q2

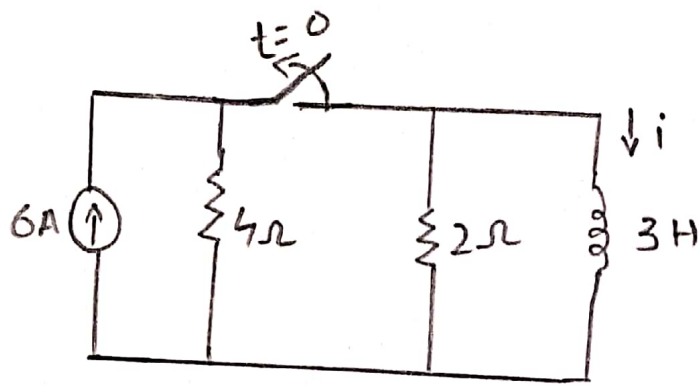


Figure 2

Solution:

For $t < 0$ the switch is closed and inductor act as ~~short~~ short circuit

Therefore inductor current $i = 6A$

For $t > 0$ the switch is opened and time constant $\tau = \frac{L}{R}$

$$\tau = \frac{3}{2}$$

Now the inductor current $i(t) = 6e^{-\frac{t}{\tau}}$

$$i(t) = 6e^{-\frac{t}{3/2}}$$

$$i(t) = 6e^{-\frac{2t}{3}} u(t) A.$$

Q No 3

Step 1

The step response of branch voltage of the given RLC circuit is describe

by

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{i}{C} = 10$$

Divide by L

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = \frac{10}{L}$$

R.H. side of equation xing by $\frac{C}{C}$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = \frac{10C}{LC}$$

And $C = 0.2F$ thus

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = \frac{2}{LC}$$

Substitute

$$\frac{d^2 i}{dt^2} + 8 \frac{di}{dt} + 10i = 20 \dots (1)$$

∴ Equation for source-free series RLC circuit is given by

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = \frac{I_s}{LC} \dots (2)$$

Step 2

Compare 1 & 2

$$\frac{R}{L} = 8 \rightarrow (3)$$

$$\frac{1}{LC} = 10 \rightarrow (4)$$

$$\frac{I_s}{LC} = 20 \rightarrow (5)$$

Step 3

From (3) α is given by

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/s} \rightarrow (6)$$

Natural frequency ω_0 is given by

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

From (4)

$$\omega_0 = \sqrt{10} \text{ rad/s} \rightarrow (7)$$

From (6) & (7)

$$\because \alpha > \omega_0$$

\therefore The circuit is overdamped

Root of characteristic eqn are given

$$\begin{aligned} s_1 &= -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\ &= -4 + \sqrt{4^2 - 10} \\ &= -4 + \sqrt{6} \text{ rad/s} \end{aligned}$$

$$\begin{aligned} &2\alpha \\ &\sqrt{4^2 - 10} \\ &\sqrt{4^2 - 10} \\ &\sqrt{16 - 10} \\ &\sqrt{6} \end{aligned}$$

From (5) steady state current

is given by

$$I_s = 20 \times LC$$

$$20 \times 0.5 \times 0.2 = 2A \rightarrow (8)$$

Step (4)

Current for overdamped case is given by

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0 \rightarrow (9)$$

Substitute $t = 0$,

$$i(0) = I_s + A_1 + A_2$$

Substitute,

$$1 = 2 + A_1 + A_2$$

Thus

$$A_1 + A_2 = -1 \rightarrow (10)$$

Step (5)

From (9) Find $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = A_1 s_1 + A_2 s_2$$

Substitute the value

$$(-4 + \sqrt{6}) A_1 + (-4 - \sqrt{6}) A_2 = 0 \rightarrow (11)$$

Solve (10) & (11) simultaneously

$$A_1 = -1.316$$

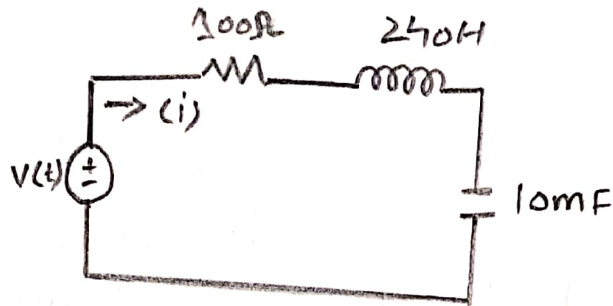
$$A_2 = 0.316$$

Step (6)

$$i(t) = 2 - 1.316 e^{(-4 + \sqrt{6})t} + 0.316 e^{(-4 - \sqrt{6})t} \quad A$$

Q4

A series of RLC Circuit
 Find the current flowing through the circuit.



Consider the following voltage is applied the series RLC circuit.

$$v(t) = 10 \cos 2t \text{ V}$$

Here

$$\text{Amplitude } V_m = 10 \text{ V}$$

$$\text{Angular Frequency } \omega = 2 \text{ rad/s}$$

$$\text{Phase Angle } \phi = 0^\circ$$

So phase for voltage, $v(t)$

$$V(t) = 10 \angle 0^\circ \text{ V}$$

Inductive Reactance of the circuit

$$X_L = \omega L$$

$$\omega = 2 \text{ rad/s} \quad , \quad L = 240 \text{ H}$$

$$X_L = (2 \text{ rad/s})(240 \text{ H})$$

$$X_L = 480 \Omega$$

Now For capacitive reactance

$$X_c = \frac{1}{\omega C}$$

~~X_c =~~ $\omega = 2 \text{ rad/s}$, $C = 10 \text{ mF}$

$$X_c = \frac{1}{2(10 \times 10^{-3})} \rightarrow$$

Bases same power
will be added

$$\Rightarrow \frac{1}{2(10^{+1-3})}$$

$$\Rightarrow \frac{1}{2(10^2)}$$

$$\Rightarrow \frac{1 \times 10^2}{2} \Rightarrow \frac{100}{2} = 50$$

$$X_c = 50 \Omega$$

Now For impedance of the circuit.

$$Z = R + jX_L - jX_c$$

$$R = 100 \Omega, X_L = 480 \Omega, X_c = 50 \Omega$$

$$\Rightarrow (100 + 480 - 50) \Omega$$

$$Z \Rightarrow (100 + j430) \Omega$$

Represent the impedance Z is
Phasor Form.

$$\begin{aligned}
 Z &= (100 + j430) \Omega \\
 &= \sqrt{(100)^2 + (430)^2} \angle \tan^{-1} \left(\frac{430}{100} \right) \\
 &= \sqrt{10000 + 184900} \angle \tan^{-1} (4.3) \\
 &= \sqrt{194900} \angle \tan^{-1} (4.3) \\
 &= 441.47 \angle 76.9^\circ \Omega
 \end{aligned}$$

Now the current flowing is

$$i = \frac{V(t)}{Z}$$

$$V(t) = 10 \angle 0^\circ, Z = 441.47 \angle 76.9^\circ$$

$$i = \frac{10 \angle 0^\circ \text{ V}}{441.47 \angle 76.9^\circ \Omega}$$

$$= \frac{10}{441.47} \angle [0 - 76.9] \text{ A}$$

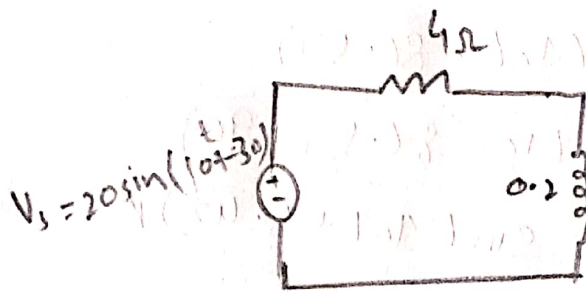
$$= 22.6 \times 10^{-3} \angle -76.9^\circ \text{ A}$$

$$= 22.6 \angle -76.9^\circ \text{ mA}$$

So the general expression for current, i .

$$i = 22.6 \cos(2t - 76.9^\circ) \text{ mA.}$$

Q No 5



$$V_s = 20 \sin(10t + 30) \text{ V}, \quad R = 4\Omega, \quad L = 0.2 \text{ H}$$

$$V_s = 20 \sin(10t + 30) \text{ V}$$
$$20 \cos(10t + 30^\circ - 90^\circ) \text{ V}$$

$$20 \cos(10t - 60^\circ) \text{ V}$$

$$V_s = 20 \angle -60^\circ \text{ V}$$

$$\omega = 10 \text{ rad/sec}$$

$$X_L = j\omega L$$

$$0.2 \text{ H} = j \times 10 \times 0.2$$

$$0.2 \text{ H} = j2\Omega$$

Now

$$Z = 4 + j2\Omega$$

$$i = \frac{20 \angle -60^\circ}{4 + j2}$$

$$I = \frac{20 \angle -60^\circ}{\sqrt{4^2 + 2^2} \angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$\underline{I} = \frac{20 \angle -60^\circ}{4.72 \angle 26.57^\circ}$$

$$I = 4.472 \angle -86.57^\circ$$

Now

$$i(t) = 4.472 \cos(10t - 86.57)$$

$$i(t) = 4.472 \sin(10t - 86.57 + 90)$$

$$i(t) = 4.472 \sin(10t + 3.43^\circ) \text{ A}$$

As

$$V = j2 \times (4.47 \angle -86.57)$$

$$V = j2 \times (0.2675 - j4.464)$$

$$V = 8.92 + j0.53512$$

From Rectangular to Polar Form

$$V = \sqrt{(8.926)^2 + (0.53512)^2} \angle \tan^{-1} \left(\frac{0.5312}{8.928} \right)$$

$$V = 8.944 \angle 3.4^\circ \text{ V}$$

Now

$$V(t) = 8.944 \cos(10t + 3.4)$$

$$V(t) = 8.944 \sin(10t + 3.4 + 90^\circ)$$

$$V(t) = 8.944 \sin(10t + 93.4) \text{ V}$$