CALCULUS AND

ANALYTICAL

GEOMETRY

Examination: Final Paper

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Paper:- Final Term.

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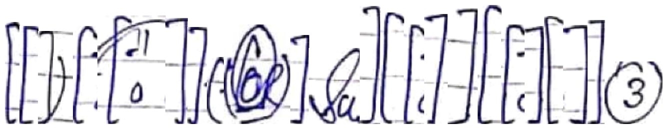

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O " '

/-IO = “’ ”

(c) Sol

Req; $X = ?$

GIVEN:- $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

Put $I = (\text{Identity matrix})_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$X + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Subtracting $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ o.b.s.

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Or $X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \left(-\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} + \left(-\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$X + \begin{bmatrix} \cancel{2}+(\cancel{-2}) & 0+0 \\ 0+0 & \cancel{2}+(\cancel{-2}) \end{bmatrix} = \begin{bmatrix} 3+(-2) & -1+0 \\ 1+0 & \cancel{2}+(\cancel{-2}) \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3-2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$X + O = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}$$

RC

Naoj P.f.V.

$$A^2+BC=\begin{bmatrix}9 & 8\\4 & 9\end{bmatrix}+\begin{bmatrix}-3 & 4\\4 & 0\end{bmatrix}$$

$$=\begin{bmatrix}9+(-3) & 8+4\\4+4 & 9+0\end{bmatrix}$$

$$=\begin{bmatrix}9-3 & 12\\8 & 9\end{bmatrix}$$

$$\boxed{A^2+BC=\begin{bmatrix}6 & 12\\8 & 9\end{bmatrix}}$$

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Q2) Sol (a)

Req; Integral $= I = ?$

GIVEN:- Let $I = \int \frac{1}{\sqrt{x^5}} dx$

(OR)

$$I = \int \frac{1}{(x^5)^{1/2}} dx.$$

$$= \int \frac{1}{x^{5/2}} dx$$

(OR)

$$= \int x^{-5/2} dx.$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C.$$

$$I = \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + C \quad \therefore -\frac{5}{2}+\frac{1}{1} = \frac{-5+2}{2} = -3/2$$

$$I = \frac{x^{-3/2}}{-3/2} + C$$

(OR) $I = \frac{-2}{3} x^{-3/2} + C$

(OR)

$$I = \frac{-2}{3x^{3/2}} + C.$$

$$I = \frac{-2}{3(x^3)^{1/2}} + C$$

(OR)

$$\boxed{I = \frac{-2}{3\sqrt{x^3}} + C.}$$

Ans

(9)

(b) Sol

Req; Integral $= I = ?$

GIVEN:- Let

$$I = \int \frac{1}{(8x+7)^8}\, dx$$

(OR)

$$I = \int (8x+7)^{-8}\, dx \longrightarrow (1)$$

Let

$$U = 8x+7 \longrightarrow (2)$$

Differentiating w.r.t x o.b.s.

$$\frac{dU}{dx} = \frac{d}{dx}(8x+7)$$

$$= \frac{d}{dx}8x + \frac{d}{dx}7$$

$Cf \xrightarrow{d/dx} c\, d/dx\, f$ $\quad C \xrightarrow{d/dx} 0$

$$= 8\frac{d}{dx}x + 0$$

(1)

$$\frac{dU}{dx} = 8$$

x

(OR) $du = 8\,dx$. (10)

$$\boxed{\frac{du}{8} = dx} \longrightarrow (3)$$

Put eq (2) & (3) in eq (1) $\Rightarrow$

$$I = \int u^{-8} \cdot \frac{du}{8}$$

$$\int cf\,dx = c\int f\,dx$$

$$I = \frac{1}{8}\int u^{-8}\,du$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$I = \frac{1}{8}\left[\frac{u^{-8+1}}{-8+1}\right] + C$$

$$I = \frac{1}{8}\left[\frac{u^{-7}}{-7}\right] + C$$

$$I = \frac{1}{-56}\,u^{-7} + C$$

(OR) (11)

$$I = \frac{-1}{56} U^{-7} + C.$$

(OR) $$I = \frac{-1}{56U^{7}} + C$$

Put $U = 8x+7 \longrightarrow$ eq (2) = 0

$$\boxed{I = \frac{-1}{56(8x+7)^{7}} + C.}$$

Q1) a) Sol

$\frac{d}{dx} f(x) = ?$

GIVEN:- Let

$$f(x) = \frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

Differentiating w.r.t x o.b.s)

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left[\frac{3x^4 - 2x^3 + 5}{x^3 + 1}\right]$$

Quotent Rule.

$$\frac{U}{V} \xrightarrow{d/dx} \frac{V \cdot \frac{d}{dx} U - U \cdot \frac{d}{dx} V}{V^2}$$

$$f'(x) = \frac{(x^3+1)\frac{d}{dx}(3x^4 - 2x^3 + 5) - (3x^4 - 2x^3 + 5) \cdot \frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)\left[\frac{d}{dx} 3x^4 - \frac{d}{dx} 2x^3 + \frac{d}{dx} 5\right] - (3x^4 - 2x^3 + 5)\left[\frac{d}{dx} x^3 + \frac{d}{dx} 1\right]}{(x^3+1)^2}$$

0 $\qquad$ 0

$$cf \xrightarrow{d/dx} c\frac{d}{dx} f$$

(13)

$$=\frac{(x^3+1)\left[3\frac{d}{dx}x^4-2\frac{d}{dx}x^3+0\right]-(3x^4-2x^3+5)\left[\frac{d}{dx}x^3+0\right]}{(x^3+1)^2}$$

Power rule

$$x^n \xrightarrow{d/dx} nx^{n-1}\cdot\frac{d}{dx}x$$

$$=\frac{(x^3+1)\left[3\left(4x^{4-1}\cdot\cancel{\frac{d}{dx}x}\right)-2\left(3x^{3-1}\cdot\cancel{\frac{d}{dx}x}\right)\right]-(3x^4-2x^3+5)\left[3x^{2-1}\cdot\cancel{\frac{d}{dx}x}\right]}{(x^3+1)^2}$$

$$=\frac{(x^3+1)\left[12x^3-6x^2\right]-\left[(3x^4-2x^3+5)(3x^2)\right]}{(x^3+1)^2}.$$

$$=\frac{(x^3+1)(12x^3-6x^2)-3x^2(3x^4-2x^3+5)}{(x^3+1)^2}.$$

Simplify

$$=\frac{(12x^6-6x^5+12x^3-6x^2)-(9x^6-6x^5+15x^2)}{(x^3+1)^2}$$

$$=\frac{12x^6-\cancel{6x^5}+12x^3-6x^2-9x^6+\cancel{6x^5}-15x^2}{(x^3+1)^2}$$

[By Re- arranging]

(14)

$$= \frac{12x^6 - 9x^6 + 12x^3 - 6x^2 - 15x^2}{(x^3+1)^2}$$

$$= \frac{3x^6 + 12x^3 - 21x^2}{(x^3+1)^2}$$ (Common)

$$\boxed{f'(x) = \frac{3x^2(x^4+4x-7)}{(x^3+1)^2}}$$

(15)

(b) Sol

Req; $\frac{d}{dx} f(x) = ?$

GIVEN:- Let

$$f(x) = \frac{(x^3+1)^2}{x^3-1}$$

1st Simplifying:-

$(a+b)^2 = a^2+b^2+2ab$

$$f(x) = \frac{(x^3)^2+(1)^2+2(x^3)(1)}{x^3-1}$$

$$f(x) = \frac{x^6+2x^3+1}{x^3-1}$$

Differentiating w.r.t x o.b.s

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{x^6+2x^3+1}{x^3-1}\right)$$

Quotient Rule

$$\frac{U}{V} \xrightarrow{d/dx} \frac{V\cdot\frac{d}{dx}U - U\cdot\frac{d}{dx}V}{V^2}$$

$$f'(x) = \frac{(x^3-1)\frac{d}{dx}(x^6+2x^3+1) - (x^6+2x^3+1)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}.$$

$$f'(x) = \frac{(x^3-1)\left[\frac{d}{dx}x^6 + \frac{d}{dx}2x^3 + \frac{d}{dx}1\right] - (x^6+2x^3+1)\left[\frac{d}{dx}x^3 - \frac{d}{dx}1\right]}{(x^3-1)^2}$$

$$cf \xrightarrow{d/dx} c\frac{d}{dx}f \qquad c \xrightarrow{d/dx} 0 \qquad c \xrightarrow{d/dx} 0$$

$$f'(x) = \frac{(x^3-1)\left[\frac{d}{dx}x^6 + 2\frac{d}{dx}x^3 + 0\right] - (x^6+2x^3+1)\left[\frac{d}{dx}x^3 - 0\right]}{(x^3-1)^2}.$$

Power rule.

$$x^n \xrightarrow{d/dx} n x^{n-1} \cdot d/dx\, x.$$

$$f'(x) = \frac{(x^3-1)\left[6x^{6-1}\cdot\cancel{\frac{d}{dx}x} + 2\left(3x^{3-1}\cdot\cancel{\frac{d}{dx}x}\right)\right] - (x^6+2x^3+1)\left[3x^{3-1}\cdot\cancel{\frac{d}{dx}x}\right]}{(x^3-1)^2}.$$

$$= \frac{(x^3-1)\left[6x^5+6x^2\right] - (x^6+2x^3+1)(3x^2)}{(x^3-1)^2}.$$

Simplify

$$= \frac{(6x^8 + \cancel{6x^5} - \cancel{6x^5} - 6x^2) - (3x^8 + 6x^5 + 3x^2)}{(x^3-1)^2}$$

$$= \frac{6x^8 - 6x^2 - 3x^8 - 6x^5 - 3x^2}{(x^3-1)^2}.$$

$5x^8 - 6x^5 - 9x^2$ — (Comma

(17)

(By Re- ng g)

$6x3x^8 - 6x6x$

$(x^3-1$

$(x^3-1)^2.$

(OR)

$$\boxed{f'(x) = \frac{x^2\left[5x^6 - 6x^3 - 9\right]}{(x^3-1)^2}}$$

D

Q3) a) Sol

Req; $I = ?$

method:- Partial fraction.

GIVEN:- Let:-

$$I = \int \frac{-x+9}{2x^2-8x+6}\,dx \longrightarrow (1)$$

By taking:-

$$= \frac{-x+9}{2x^2-8x+6} \longrightarrow (2)$$

As Degree of (N) < Degree of (D)

$$1 < 2.$$

It is proper rational fraction.

Now make factors of:-

$$= 2x^2-8x+6$$

By sum & product

$$= 2x^2 - \boxed{6x} - \boxed{2x} + 6$$

(common)

$$= 2x(x-3) - 2(x-3)$$

(common)

(19)

$= (x-3)(2x-2)$ (common)

$= \boxed{2(x-3)(x-1)}$

Now; (2) $\Rightarrow$ becomes.

$$\frac{-x+9}{2x^2-8x+6} = \frac{-x+9}{2(x-3)(x-1)} = \frac{1}{2}\left[\frac{-x+9}{(x-3)(x-1)}\right] \longrightarrow (3)$$

Let;

$$\frac{-x+9}{(x-3)(x-1)} = \frac{A}{x-3} + \frac{B}{x-1} \longrightarrow (4)$$

Xing $(x-3)(x-1)$ o.b.s. $\Rightarrow$

$$-x+9 = A(x-1) + B(x-3) \longrightarrow (5)$$

Put $x-3=0$ in eq (5) $\Rightarrow$

$\boxed{x=3}$

$$-3+9 = A(3-1) + B(\cancel{3}-\cancel{3})$$

$$6 = A(2) + 0$$

$$6 = 2A$$

$\Rightarrow 2A = 6$

$A = \cancel{6}/\cancel{2}^{3}$

$$\boxed{A = 3}$$

Now put $x-1=0 \Rightarrow x=1$ in eq (5) $\Rightarrow$

$-1+9 = A(\cancel{1-1}^{0}) + B(1-3)$

$8 = 0 + (-2B)$

$8 = -2B$

$\Rightarrow B = \cancel{8}^{4}/-\cancel{2}$

$$\boxed{B = -4.}$$

Put v of (A) & (B) in eq (4) $\Rightarrow$

$$\frac{-x+9}{(x-3)(x-1)} = \frac{3}{x-3} + \frac{(-4)}{x-1}$$

$$= \boxed{\frac{3}{x-3} - \frac{4}{x-1}} \longrightarrow (6)$$

(21)

Put eq (6) in eq (3) $\Rightarrow$

$$= \boxed{\frac{1}{2}\left[\frac{3}{x-3} - \frac{4}{x-1}\right]} \quad \text{--- (7)}$$

Put eq (7) in eq (1) $\Rightarrow$

$$I = \int \frac{1}{2}\left[\frac{3}{x-3} - \frac{4}{x-1}\right] dx.$$

$$\int cf \xrightarrow{d/dx} c\int f\,dx$$

$$I = \frac{1}{2}\int\left(\frac{3}{x-3} - \frac{4}{x-1}\right) dx$$

(or)

$$I = \frac{1}{2}\left[\int \frac{3}{x-3}\,dx - \int \frac{4}{x-1}\,dx\right].$$

$$\int cf\,dx = c\int f\,dx$$

$$= \frac{1}{2}\left[3\int \frac{1}{x-3}\,dx - 4\int \frac{1}{x-1}\,dx\right]$$

$$\int \frac{1}{x} = \ln x + c$$

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$$= \frac{1}{2}\left[3\ln|x-3| - 4\ln|x-1|\right] + C$$

$$n\ln m = \ln m^n$$

$$\boxed{= \frac{1}{2}\left[\ln|x-3|^3 - \ln|x-1|^4\right] + C}$$

$$\ln m - \ln n = \ln \frac{m}{n}$$

$$\boxed{= \frac{1}{2}\left[\ln \frac{|x-3|^3}{|x-1|^4}\right] + C}$$

(OR) $$n\ln m = \ln m^n$$

$$\boxed{= \left[\ln\left|\frac{(x-3)^3}{(x-1)^4}\right|\right]^{1/2} + C}$$

(23)

(b) Sol

Req; $I = ?$

method:- Partial fraction

Given:- let

$$I = \int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx \longrightarrow (1)$$

By taking:-

$$= \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} \longrightarrow (2)$$

As Degree of (N) < Degree of (D)

2 < 4.

It is Proper rational fraction.

eq (2) $\Rightarrow$

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3} \longrightarrow (3)$$

Xing $(x^2+1)(x^2+2x+3)$ o.b.s.

we get:-

$$4x^2+8x = (Ax+B)(x^2+2x+3) + (Cx+D)(x^2+1)$$

$$= Ax^3+2Ax^2+3Ax+Bx^2+2Bx+3B+Cx^3+Cx+Dx^2+D$$

(By Re-arranging)

$$\boxed{4x^2+8x = Ax^3+Cx^3+2Ax^2+Bx^2+Dx^2+3Ax+2Bx+Cx+3B+D}$$

(Common)

$$4x^2+8x = (A+C)x^3 + (2A+B+D)x^2 + (3A+2B+C)x + (3B+D) \quad \text{—(4)}$$

By Compairing co-efficients

For x^3:- $\quad x^3 = x^3$

$$\boxed{A+C=0} \text{ —o (i)}$$

For x^2:- $\quad x^2 = x^2$

$$\boxed{2A+B+D=4} \text{ —o (ii)}$$

For x:- $\quad \boxed{3A+2B+C=8} \text{ —o (iii)}$

For Constt:- $\quad 3B+D=0 \text{ —o (iv)}$

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Subtracting (iii) & (iv)

$$2A + B + \cancel{D} = 4.$$

$$(-)\quad 3B + \cancel{D} = 0.$$

$$2A - 2B = 4.$$

$$2(A - B) = 4.$$

$$A - B = \cancel{4}/\cancel{2}$$

$$\boxed{A - B = 2.} \longrightarrow (V)$$

Now put $A + C = 0 \Rightarrow C = -A$ in eq (iii) $\Rightarrow$

$$3A + 2B + (-A) = 8.$$

$$3A - A + 2B = 8$$

$$2A + 2B = 8$$

$$2(A + B) = 8.$$

$$A + B = \cancel{8}/\cancel{2}$$

$$\boxed{A + B = 4} \longrightarrow (Vi)$$

Adding eq (V) & eq (Vi) we get :-

$$A - \cancel{B} = 2.$$

$$(+)\quad A + \cancel{B} = 4.$$

$$2A = 6$$

$$2A = 6$$

$$A = \cancel{6}/\cancel{2}$$

$$\boxed{A = 3}$$

eq (v) $\Rightarrow A - B = 2.$

$$\Rightarrow B = A - 2 \quad \text{put } A = 3$$

$$B = 3 - 2$$

$$B = 1$$

put $A = 3$ & $B = 1$ in eq (iii) $\Rightarrow$

$$3(3) + 2(1) + C = 8.$$

$$9 + 2 + C = 8$$

$$11 + C = 8$$

$$C = 8 - 11$$

$$\boxed{C = -3}$$

put $A = 3$ & $B = 1$ in eq (ii) $\Rightarrow$

$$2(3) + 1 + D = 4.$$

$$6 + 1 + D = 4.$$

$$7 + D = 4$$

(27)

$$D = 4-7.$$

$$\boxed{D = -3.}$$

P.t.v of (A), (B), (C) & (D) in eq (3) $\Rightarrow$

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{3x+1}{x^2+1} + \frac{(-3)x+(-3)}{x^2+2x+3}$$

$$= \boxed{\frac{3x+1}{x^2+1} - \frac{3x+3}{x^2+2x+3}}$$

Put in eq (1) $\Rightarrow$

$$I = \int\left(\frac{3x+1}{x^2+1} - \frac{3x+3}{x^2+2x+3}\right)dx$$

$$I = \int\left[\frac{3x}{x^2+1} + \frac{1}{x^2+1} - \frac{3x+3}{x^2+2x+3} - \frac{1}{x^2+2x+1+2}\right]$$

$$I = \int\frac{3x}{x^2+1}dx + \int\frac{1}{x^2+1}dx - \int\frac{3x+3}{x^2+2x+3}dx - \int\frac{1}{x^2+2x+1+2}dx$$

Xing & ÷ing by 2

$$I = \frac{3}{2}\int\frac{2x}{x^2+1}dx + \int\frac{1}{x^2+1}dx - \frac{3}{2}\int\frac{2x+2}{x^2+2x+3}dx - \int\frac{1}{x^2+2x+1+2}dx$$

$$I = \frac{3}{2}\ln|x^2+1| + \ln|x^2+1| + \frac{3}{2}\ln|x^2+2x+3| - \int \frac{1}{(x+1)^2+(\sqrt{2})^2}\,dx$$

$$n\ln m = \ln m^n$$

$$I = \ln|(x^2+1)^{3/2}| + \ln|x^2+1| + \ln|(x^2+2x+3)^{3/2}| - \frac{1}{\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}} + C$$

$$\ln m + \ln n + \ln o = \ln(m \times n \times o)$$

$$I = \ln\left|\left[(x^2+1)(x^2+2x+3)\right]^{3/2}(x^2+1)\right| - \frac{1}{\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}} + C$$

:: END :-