NAME: MOAZZAM AJMAL
ID# 16977
SUBMITTED TO : SIR ABRAR

Name: Moazzam Ajmal I.D:- 16477

Calculus and analytical geometry

Final Paper (BS, SE) Submitted to:-

Date: 27th June, 2020 Muhammad Abrar Khan

Note: Attempt all questions.

Q1) Differentiate $\frac{3x^4-2x^3+5}{x^3+1}$ with respect to X.
(a)

Sol:-

$$y=\frac{3x^4-2x^3+5}{x^3+1}$$

Diff w.r.t x.

$$\frac{dy}{dx}=\frac{(x^3+1)\,d/dx\,(3x^4-2x^3+5)-(3x^4-2x^3+5)\,d/dx\,(x^3+1)}{(x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{(x^3+1)(12x^3-6x^2)-(3x^4-2x^3+5)(3x^2)}{(x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{12x^6-\cancel{6x^5}+12x^3-6x^2-9x^6+\cancel{6x^5}-15x^2}{(x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{3x^6+12x^3-21x^2}{(x^3+1)^2}$$

$$=\frac{3x^2(x^4+4x-7)}{(x^3+1)^2}$$ Ans.

b) Differentiate $\frac{(x^3+1)^2}{x^3-1}$ with respect to X.

Sol:-

$$y=\frac{(x^3+1)^2}{x^3-1}$$

Diff w.r.t x

$$\frac{dy}{dx}=\frac{(x^3-1)\,d/dx\,(x^3+1)^2-(x^3+1)^2\,d/dx\,(x^3-1)}{(x^3-1)^2}$$

$$\frac{dy}{dx}=\frac{3x^2(x^3+1)[2(x^3-1)-(x^3+1)]}{(x^3-1)^2}$$

$$\frac{dy}{dx}=\frac{3x^2(x^3+1)[2x^3-2-x^3-1]}{(x^3-1)^2}$$

$$\frac{dy}{dx} = \frac{3x(x^3+1)(x^3-3)}{(x^3-1)^2}$$ Ans

Q2 (a) Find the integration of $\int \frac{1}{\sqrt{x^5}}\,dx$

Sol:-

$$\int \frac{1}{\sqrt{x^5}}\,dx$$

$$\int \frac{1}{(x^5)^{1/2}}\,dx$$

$$\int \frac{1}{x^{5/2}}\,dx$$

$$\int x^{-5/2}\,dx$$

$$\frac{x}{-5/2+1} + C$$

$$\frac{x^{\frac{-5+2}{2}}}{\frac{-5+2}{2}} + C$$

$$\frac{x^{-3/2}}{-3/2} + C$$

$$-\frac{2}{2}x^{-3/2} + C$$

$$-\frac{2}{3}\frac{1}{x\sqrt{x}} + C$$ Ans.

b) Find the integration of $\int \frac{1}{(8x+7)^8}\,dx$.

Sol:-

$$\int \frac{1}{(8x+7)^8}\,dx$$

$\int (8x+7)^{-8} dx$

Multiply and divide by 8.

$\frac{1}{8} \int (8x+7)^{-8} dx$

$\frac{1}{8} \frac{(8x+7)^{-8+1}}{-8+1} + C$

$\frac{1}{8} \frac{(8x+7)^{-7}}{-7} + C$

$-\frac{1}{56} (8x+7)^{-7} + C$

$-\frac{1}{56} \frac{1}{(8x+7)^7} + C$

Ans.

Q3) Find the integration of.

a) $\int \frac{-x+9}{2x^2-8x+6} dx$ by partial fractions.

Sol:-

$\int \frac{-x+9}{2x^2-8x+6} dx$

$\frac{-x+9}{2x^2-2x-6x+6} = \frac{-x-9}{2x(x-1)-6(x-1)}$

$\frac{-x+9}{(x-1)(2x-6)} = \frac{A}{(x-1)} + \frac{B}{2x-6}$

Multiply by both sides $(x-1)(2x-6)$.

$x+9 = A(2x-6) + B(x-1) - C$

$x-1=0$

$x=1$

$(-1)+9 = A(2(1)-6) + B(1-1)$

$10 = A(-4)$

$10 = -4A$

$\frac{10}{-4} = \frac{-4A}{-4}$

$\frac{-5}{2} = A$

$2x - 6 = 0$

$2x = 6$

$x = 3$

Placed in (x)

$-x + 9 = A(2x-6) + B(x-1)$

$-(3) + 9 = A(2(3)-6) + B(3-1)$

$6 = 0 + 2B$

$\frac{6}{2} = \frac{2B}{2}$

$3 = B$

$\Rightarrow \frac{-x+9}{2x^2-8x+6} = \frac{-5/2}{(x-1)} + \frac{3}{2x-6}$

$\int \frac{-x+9}{2x^2-8x+6}\,dx = \int \frac{-5}{2(x-1)} + \frac{3}{2x-6}\,dx$

$= \frac{-5}{2}\int \frac{1}{x-1}\,dx + 3\int \frac{1}{2(x-3)}\,dx$

$\int \frac{-x+9}{2x^2-8x+6}\,dx = \frac{-5}{2}\int \frac{1}{x-1} + x\ \frac{3}{2}\int \frac{1}{x-3}\,dx$

$\int \frac{-x+9}{2x^2-8x+6}\,dx = \frac{-5}{2}\ln(x-1)\ \frac{3}{2}\ln(x-3)$

Ans.

b) Find the integration of $\int \frac{4x^2+8x}{(x^2+1)(x^2+2x-3)}\,dx$ by partial fractions.

Sol:-

So, $\dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{x^2+2x+3}$

$\int \dfrac{4}{x^2+1}\,dx + \int \dfrac{-12}{x^2+2x+3}\,dx$

$\int \dfrac{4}{x^2+1}\,dx = 4\int \dfrac{1}{x^2+1}\,dx$

standard integratl for $\dfrac{1}{x^2+1} = \arctan(x)$

So

$4\int \dfrac{1}{x^2+1}\,dx$

$= 4\arctan(x) + C$

substituting u

$= \int \sqrt{2}/2u^2+2 \; du = \frac{1}{\sqrt{2}} \int \frac{1}{u^2+1} \; du$

$\int \frac{1}{u^2+1} \; du = \arctan(u)$

So ; $\frac{1}{\sqrt{2}} (\arctan(u))$

$= \arctan(x+1/\sqrt{2}) / \sqrt{2}$

∴ replacing u

$-3 \cdot 2^{3/2} \arctan\left(\frac{2x+2}{2^{3/2}}\right) + c$

$\Rightarrow -3 \cdot 2^{3/2} \arctan\left(x+1/\sqrt{2}\right) + c$

. final answer

$$\int \frac{-12}{x^2+2x+3}\, dx$$

$$-12 \int \frac{1}{x^2+2x3}\, dx$$

solving for integral

$$\int \frac{1}{x^2+2x+3}\, dx = \int \frac{1}{(x+1)^2+2}\, dx$$

$$u = \frac{x+1}{\sqrt{2}} \rightarrow \frac{du}{dx} = \frac{1}{\sqrt{2}}$$

$$\frac{du}{dx} = \frac{1}{\sqrt{2}} (x+1) \frac{d}{dx} = \frac{1+0}{\sqrt{2}}$$

$$\frac{du}{dx} = \frac{1}{\sqrt{2}}$$

Q3(b)

$$\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx$$

$$\frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+2x+3)}$$

$$4x^2+8x = Ax+B\,(x^2+2x+3) + C(x+D)(x^2+1)$$

$$4x^2+8x = Ax^3+2Ax^2+3Ax+Bx^2+2Bx+3B+Cx^3$$
$$+\ C+Dx^2+D$$

Co efficient comparison

$$0 = A+C \qquad 4 = 2A+8 \qquad 8 = 3A-2B$$
$$8 = 4A+2B \qquad 8 = 3(0)-2B$$
$$-C = A \qquad +8 = 2A-2B \qquad 8 = 2B$$
$$-C = 0 \qquad 0 = 2A \qquad 4 = B.$$
$$C = 0 \qquad 0 = A$$

$$0 = C+D+3B$$
$$0 = 0+D+3(4)$$
$$-12 = D.$$

$$\int \frac{4x^2+8x}{(x^2+1)(x^2+2x-3)}\,dx = \int \frac{4}{x^2+1}\,dx + \int \frac{-12}{x^2+1}\,dx$$

Ans

Q4 Solve each of the following.

a) Matrix equations.

a) $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

Sol:-

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5-3 & 1-(-1) \\ -3-2 & 1-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 \\ & \end{bmatrix}$$

b) $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

Sol:-

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5-0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -2+1 & -2-0 \\ -1-0 & 5-2 \end{bmatrix}$$

$$X \quad \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} \text{ Ans.}$$

c) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

Sol:-

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3-2 & -1-0 \\ 1-0 & 2-2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \text{ Ans.}$$