

Q 1 :

Ans: Solution:

$$\left[\begin{array}{ccccc} 1 & 103 & 3 & 0 & 5 \\ 0 & 1 & -103 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 103 \end{array} \right] \rightarrow \text{A}$$

Put : $103 = 0$, and $-103 = -9$ in (A)

$$\left[\begin{array}{ccccc} 1 & 0 & 3 & 0 & 5 \\ 0 & 1 & -9 & 0 & 7 \\ 0 & 0 & 1 & 1 & -6 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

Now it implies that it is in echelon form and

we can find all the variables without any row operation.

$$x_1 + 0x_2 + 3x_3 + 0x_4 = 5 \rightarrow (1)$$

$$0x_1 + x_2 + (-9)x_3 + 0x_4 = 7 \rightarrow (2)$$

$$0x_1 + 0x_2 + x_3 + 1x_4 = -6 \rightarrow (3)$$

$$0x_1 + 0x_2 + 0x_3 + 1x_4 = 0 \rightarrow (4)$$

from eq (3) (4)

$$0 + 0 + 0 + x_4 = 0$$

$$x_4 = 0$$

Date: _____

(Page : 02 , I.D : 16049)

Now from eq (3)

$$x_3 + x_4 = -6$$

Now we will put the value of x_4 in eq (3)

$$x_3 + 0 = -6$$

$$x_3 = -6$$

Now from eq (2)

$$x_2 + x_3 + x_4 = 7$$

$$x_2 + (-6) = 7$$

$$x_2 - 6 = 7$$

$$x_2 = 7 + 6$$

$$x_2 = 13$$

Now from eq (1)

$$x_1 + x_2 + x_3 + x_4 = 5$$

$$x_1 + 13 + (-6) + 0 = 5$$

$$x_1 + 13 - 6 = 5$$

$$x_1 + 7 = 5$$

$$x_1 = 5 - 7$$

\Rightarrow

$$x_1 = -2$$

Date: _____

(Page : 03)

I.D = 16049 (Section "A")

So the solution is

$$S.S \{x_1, x_2, x_3, x_4 = -2, 13, -6, 0\}$$

Ans \nearrow

Ans - 2 : Solution :

$$(A) \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Now taking "A"

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & -5 & 1 \end{bmatrix}$$

$$P \cdot T \cdot D \Rightarrow$$

Date: _____

(Page : 04 , I.D = 16049 , Section "A")

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} R_3 - 2R_2$$

Now taking "B"

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} R_3 + 2R_2$$

By comparing the Matrix A and A is equal

$$A = B$$

Ans \uparrow

Date: _____

(Page :- 05 , I.D : (16049 , Section (A)))

8)

A) Given:-

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

is in echelon form.

Solution:

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

The leading entry of the 1st row is not equal to one. All the entries in the column above and below a leading 1 are not zero. So its not the echelon form.

b) Given :-

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is in echelon form.

P.T.O \Rightarrow

Date: _____

(Page: 06 > I.D: (16049, Section "A")

a) ~~Given:-~~

Solution:-

$$\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Yes, it is Echelon form, because the first non-zero entry is 1. Number of zeros to the left side of the key entry increases row by row.

c) Given:-

$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

is in reduced row echelon form.

Solution: $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

the given matrix is not reduced row echelon form because the first non-zero number in first row (the first leading entry) is not equal to 1. and the given matrix is not Echelon form.

Date:

Page: 07

I.D: 160193 Section "A"

d) Given:

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

is in reduced row Echelon form.

Solution:-

The given matrix is not reduced row echelon form because the given matrix is not in echelon form and all the entries in non key entry column is not zero.

zero.

Ans. 03:

(A):

Difference between Echelon and Reduced row is

Echelon form:-

A matrix that has undergone Gaussian elimination is said to be in row echelon form.

Characteristic:-

- 1) All zero row are at the bottom of the matrix.
- 2) The leading entry in any non-zero row is 1.
- 3) All entries in the column above and below a leading 1 are zero.
- 4) The leading entry of each non-zero row after the first occurs to the right of the leading entry of the previous row.

Date: _____

(Page :- 08 , I.D : 16049 , Section "A").

Reduced row echelon form :- (~~Practical use of~~)

Reduced row echelon form is a type of matrix used to solve system of linear equations.

Requirements :-

- 1) The first non-zero number in the first row is the number 1.
- 2) The second row also starts with the number 1, which is further to the right than the leading entry.
- 3) The leading entry row must be the only non-zero numbers in its column.
- 4) Any non-zero rows are placed at the bottom of the matrix.

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

Practical uses of Reduced row echelon form :

The echelon form of a matrix is not unique, which means there are infinite answers are possible which you perform row reduction. Reduced row echelon form is at the other form of the spectrum, it is unique, which mean row reduction on a matrix will produced the same answer no matter how you perform the same row operation.



(Page: 09 , ID: 10009 , Section: 08)

Ques. no.:

Q.5)

Solution:

$$\left[\begin{array}{ccc|c} 1 & 10 & 6 & 8 \\ 2 & 8 & -1 & -5 \\ -1 & 0 & 0 & 0 \\ \hline 1 & -4 & -1 & 19 \end{array} \right] \text{ (RHS = 19)}$$

Let:-

$$\text{(Put } 10x = 6, -10z = -0, -10y = \text{last} = 19)$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 8 & 8 \\ 2 & 8 & -1 & -1 \\ -1 & 0 & 0 & 0 \\ \hline 1 & -4 & -1 & 19 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 8 & 8 \\ 2 & 8 & -1 & -1 \\ 1 & -4 & 1 & 19 \\ \hline 0 & 0 & 0 & 0 \end{array} \right] \quad R_3 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 8 & 8 \\ 0 & -4 & -17 & -17 \\ 1 & -4 & 1 & 19 \\ \hline 0 & 0 & 0 & 0 \end{array} \right] \quad R_2 = 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 8 & 8 \\ 1 & -4 & 1 & 19 \\ 0 & -4 & -17 & -17 \\ \hline 0 & 0 & 0 & 0 \end{array} \right] \quad R_2 \leftrightarrow R_3$$

(P.T.O) \rightarrow 10 marks

Page : 10

(Page : 10 ; I.D : 16049 , Section "A")

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -10 & 22 \\ 0 & -4 & -17 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 - R_1$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -40 & 44 \\ 0 & -40 & -170 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} 4 \times R_2, \\ \text{WR } 10 \times R_3 \end{array}$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -40 & 44 \\ 0 & 0 & -214 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 - R_2$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 0 & -40 & -44/40 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} -R_2/40 \\ R_3/214 \end{array}$$

$$S.S = \left\{ \begin{bmatrix} 1 & 6 & 8 \\ 0 & 1 & -44/40 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \right.$$

Ans \nearrow

Date: _____

~~Page 1 of 1~~

Name : Abbas Khan

I.D : 16049

Section : "A"

Deptt : Software Engineering

Subject : Linear Algebra

Submitted to : "Shakeel Sir"