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Name : M. Daud
ID : 7769

Subject : Applied Calculas

Submitted to:
Engr : Shomaila mazhar

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Qno 1

Sol:

Coordinate of $P = (4, 1, 3)$

$OP = 4i + 1j + 3k$

or $OQ = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k \rightarrow (1)$

Now distance between P and Q $= |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11} \rightarrow (2)$ $d = 3.3167$

Let M be the point which ÷ PQ in ratio 1:3, then by ratio theorem

Position vector of $M = \overrightarrow{OM}$

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$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \longrightarrow (3)$$

hence eq (1), (2) and (3) are the required solution.

Q.no 2

Evaluate $\int \frac{4x^3 + 10x + 4}{2x^2 + x} dx$

Solution :: $\int \frac{4x^3 + 10x + 4}{2x^2 + x}$

$= \int \frac{2\ (2x^3 + 5x + 2)}{x\ (2x+1)} dx$

Apply linearity

$= 2 \int \frac{2x^3 + 5x + 2}{x\ (2x+1)} dx$

Now

$\int \frac{2x^3 + 5x + 2}{x\ (2x+1)} dx$

Perform polyonomial long division

$= \int \left(\frac{11x + 4}{2x\ (2x+1)} + \frac{2x - 1}{2} dx \right.$

Apply linearity

$= \frac{1}{2} \int \frac{11x + 4}{x(2x+1} dx + \int x\, dx - \frac{1}{2} \int dx$

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Now $\int \frac{11x+4}{x(2x+1)} dx$

partial fraction decomposition

$$= \int \left(\frac{3}{2x+1} + \frac{4}{x} \right) dx$$

$$= 3 \int \frac{1}{2x+1} dx + 4 \int 1/x \, dx$$

Solving

$$\int \frac{1}{2x+1} dx$$

Substitute $u = 2x+1$ $\frac{du}{dx} = 2$ $dx = 1/2 \, du$

$$= 1/2 \int 1/u \, du$$

Solving

$$\int 1/u \, du$$

This is standard Integral

$$= \ln(u)$$

$$1/2 \int 1/u \, du$$

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put again $u = 2x + 1$

$$= \frac{\ln(2x+1)}{2}$$

Now solving $\int \frac{1}{x} dx$

use previous answer

$$= \ln(x)$$

$$3 \int \frac{1}{2x+1} dx + 4 \int \frac{1}{x} dx$$

$$= \frac{3\ln(2x+1)}{2} + 4\ln(x)$$

Now $\int x \, dx$

Apply power rule

$$= \frac{x^2}{2}$$

Now $\int 1 \, dx$

Apply constant rule

$$= x$$

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$$\frac{1}{2}\int \frac{11x+4}{x(2x+1)}\,dx + \int x\,dx - \frac{1}{2}\int 1\,dx$$

$$= \frac{3\ln(2x+1)}{4} + 2\ln(x) + \frac{x^2}{2} - \frac{x}{2}$$

$$= \frac{3\ln(2x+1)}{4} + 4\ln(x) + x^2 - x$$

Add constant to solution

$$= 3\ln(|2x+1|) + 4\ln(|x|) + x^2 - x + C$$

$$= 3\ln(|2x+1|) + 4\ln(|x|) + (x-1)x + C$$

$$= x^2 - x + \frac{3}{2}\ln|2x+1| + 4\ln|x| - \frac{3}{4} + C.$$

Ans.

Q NO 3 (a) $\int_0^2 x^2 e^x \, dx$

(a) First we find Integration.

$$= \int x^2 e^x \, dx$$

$$= x^2 \int e^x dx - \int (e^x dx \, \frac{d}{dx} x^2) \, dx$$

$$= x^2 e^x - \int e^x (2x) \, dx$$

$$= x^2 e^x - 2 [x \int e^x dx - \int e^x dx \, \frac{d}{dx} x) \, dx]$$

$$= x^2 e^x - 2 [x e^x - \int e^x dx)$$

$$= x^2 e^x - 2x e^x + 2e^x$$

Put limits.

$$= \left| x^2 e^x - 2x e^x + 2x^x \right|_0^2$$

$$= (2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0))$$

$$= (4\cancel{e^2} - 4\cancel{e^2} + 2e^2 - 2$$

$$= \boxed{2e^2 - 2} \text{ Ans.}$$

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Q no 3

b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

first find the integration.

$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ?$ — ①

let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\boxed{2dy = \frac{1}{\sqrt{x}}\,dx}$ put in eq ①

$\int \sin(y)\,(2dy) = 2\int \sin(y)\,dy$

$= 2(-\cos y)$

$= -2\cos y$

put $y = \sqrt{x}$

$= -2\cos\sqrt{x}$

put limits.

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$= -2 \left| \cos\sqrt{x} \right|_1^2 = -2\left(\cos\sqrt{2} - \cos 1\right)$

$= -2\cos\sqrt{2} + 2\cos(1)$

Ans.

Q No 4

The laplace equation in 3.d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (A)$$

So $u(x, b, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$

$$u(x,y,z) = (x^2 + y^2 + z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}} (x^2 + y^2 + z^2)^{-3/2} (\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x (x^2 + y^2 + z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/\cancel{2}) (x^2 + y^2 + z^2)^{-5/2} (\cancel{2}x) + (x^2 + y^2 + z^2)^{-3/2} \right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2 (x^2 + y^2 + z^2)^{-5/2} (\cancel{}) (x^2 + y^2 + z^2)^{-3/2} \quad — (1)$$

Now

$$\frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}} (x^2 + y^2 + z^2)^{-3/2} (\cancel{2}y)$$

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$$\frac{\partial u}{\partial y} = -y\,(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = \cancel{3y}\,\cancel{(x^2+y^2+z^2)^{-5/2}} - \cancel{(x^2+y^2+z^2)^{-3/2}}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial u}{\partial z} = \frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z\,(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} \quad \text{—③}$$

put 1, 2, 3 in eq (A)

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$$= \Big(3x^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2 (x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \Big)$$

$$= (x^2+y^2+z^2)^{-5/2} \Big[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2) \Big]$$

$$= (x^2+y^2+z^2)^{-5/2} \Big[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - z^2 + 3y^2 - \cancel{x^2} - \cancel{y^2} - z^2 + 3z^2 - \cancel{x^2} - \cancel{y^2} - z^2 \Big)$$

$$= (x^2+y^2+z^2)^{-5/2} (0) = 0$$

So given $u(x, y, z)$ is the solution of Laplace equation.