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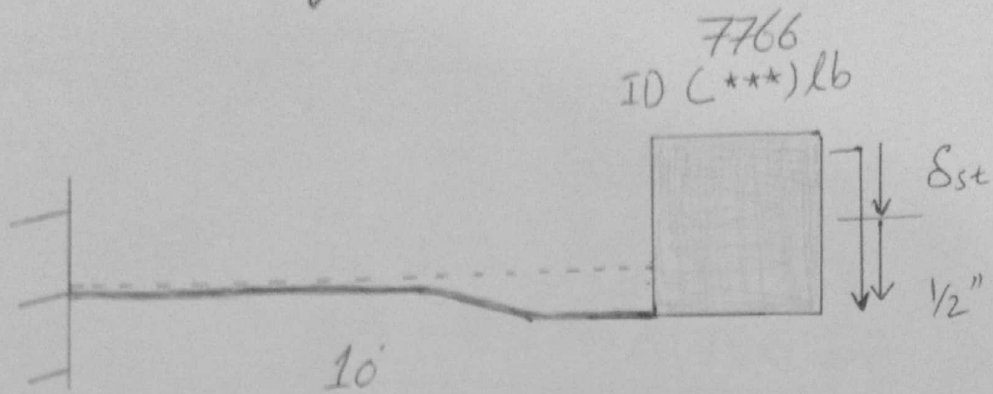
Instructor ≈ Eng. Yaseen Mahmood

Subject ≈ Intro To Structural
Dynamic & Earth Quak.

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IQRA National University

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Q#



Figure

Ans:

Given Data:

$$E = 29,000 \text{ Ksi}$$

$$I = 150 \text{ in}^4$$

$$\delta_{st} = 7766$$

Solution:

The general E.O.M for S.D.F systems is,

$$Ku + cu + mu = P(t) \quad \text{---(i)}$$

in our case system is undamped ($c=0$) under going free vibration

($P(t)=0$) Hence general EOM becomes

$$Ku + mu = 0 \quad \text{---(ii)}$$

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$$K = 3EI/L^3 \quad \text{So put value}$$

$$= \frac{3 \times 29000 \text{ ksi} \times 150 \text{ in}^4}{10 \times 12 \text{ in}}$$

$$K = 7.55 \text{ k/in}$$

To eliminate mistake during calculation use fundamental units like lb, ft, sec etc.

$$\text{So } K = 7.55 \text{ k/in} = \boxed{96025 \text{ lb/ft}}$$

$$m = \frac{7766 \text{ lb sec}^2}{32.2 \text{ ft}}$$

$$\boxed{m = 241.180 \text{ slug}}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{96025}{241.180}}$$

$$\omega_n = 19.953 \text{ rad/sec}$$

P.T.O

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$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.953} = \boxed{0.314 \text{ sec.}}$$

Substituting the corresponding values
in eq (ii)

$$90625u + 241.180 = 0$$

where "K" is the lb/ft and
"m" is in ~~lb~~ lb sec/ft²

General solution to EOM for
undamped free vibration is ;

$$u(t) = u(0) \cos(\omega_n t) + \frac{u'(0)}{\omega_n} \sin(\omega_n t)$$

$$u(0) = \frac{1}{2} = \frac{1}{24} \text{ ft and } u'(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.9t) + 0 =$$

$$\text{equivalent, } \left(\frac{1}{24}\right) \times \cos(19.9t)$$

"t" is

$$f_s(t) = K \cdot u(t) = \frac{90625 \times \cos(19.9t)}{24}$$

$$f_s(t) = 7552.08 \times \cos(19.9t) \quad \text{P.T.O}$$

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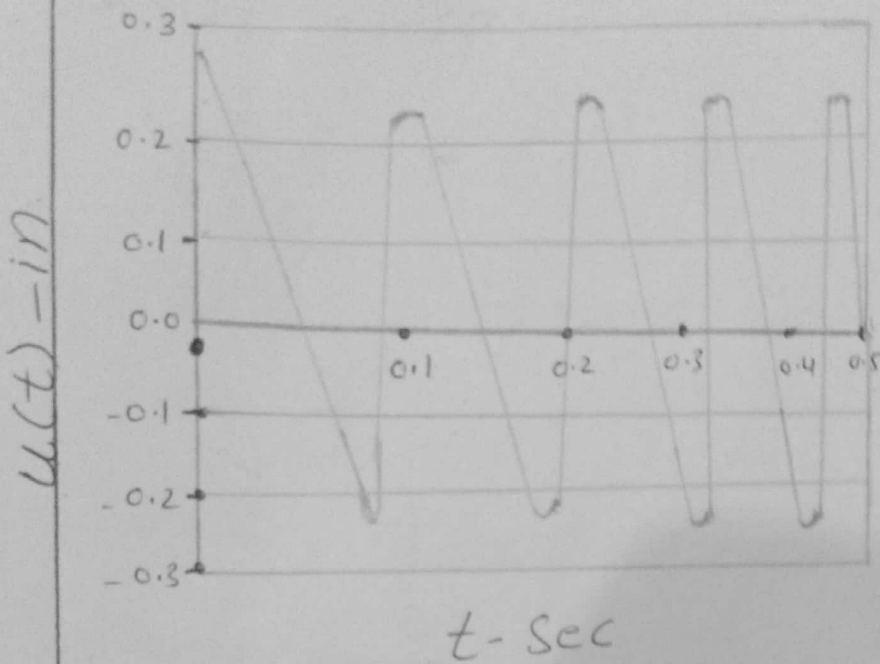
Amplitude of dynamic displacement u_0 for undamped free vibrations is;

$$u_0 = \sqrt{[u(\omega)^2 + (u(\omega)/\omega n)^2]} = \sqrt{(1/24)^2 + 0}$$
$$= \frac{1}{24} \text{ ft}$$

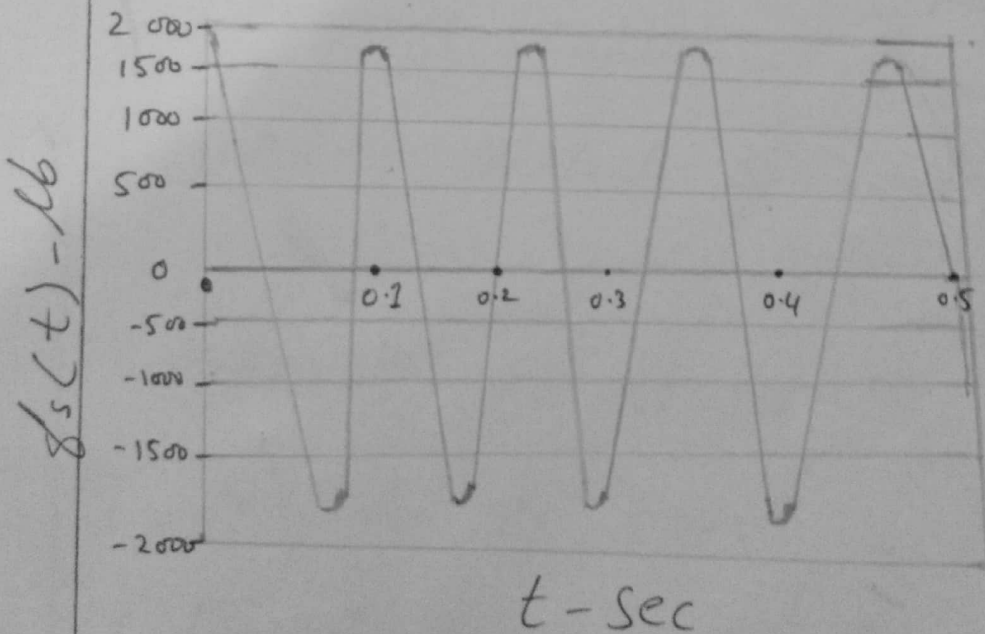
Amplitude of equivalent static force, is f_{s0}

$$K u_0 = 90025 \times \frac{1}{24}$$
$$= \boxed{3776.04 \text{ Ans}}$$

Undamped Free Vibration



⇒ Vibration of displacement with time.



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OH

Problem: 3

Given Data:

- Force = 60 kips
- $u_1 = \frac{7766}{1000} = 7.766$
- After $j = 7$ (cycles)
- Completed = 3.57 sec

Required Data:

- (a) Damping ratio
- (b) Natural period of undamped vibration
- (c) stiffness of structure
- (d) weight of tank
- (e) Damping coefficient
- (f) No of cycles to reduce the displacement amplitude to 0.5.

Sol.

$$u_1 = 7.7''$$

attd. $j=7, u_{j+1} = u_8 = 0.9''$

(a) Damping Ratio = $\zeta = ?$

$$\zeta = \frac{1}{2\pi\zeta} \ln \left[\frac{u_1}{u_{j+1}} \right]$$

$$\zeta = \frac{1}{2\pi\zeta} \ln \left(\frac{7.7}{0.9} \right)$$

$$\zeta = 0.0488 = 4.88\%$$

(b) $T_n = ?$

7 cycles of vibration are completed in 3.57 seconds

⇒ Time required to complete one cycle = $\frac{3.57}{7}$

$$T_0 = 0.51 \text{ Sec}$$

Now,

$$\omega_n = \omega_n \sqrt{(1-\delta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2K}{(\omega_n \sqrt{1-\delta^2})}$$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{1-\delta^2}}$$

$$T_n = T_D \times \sqrt{1-\delta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.488)^2}$$

$$\Rightarrow T_n = 0.445 = 0.46 \text{ sec.}$$

(C) stiffness of structure = $K = ?$

$$K = \frac{F \cdot \cos \theta}{7.766} = \frac{60 \cos 60}{7.766}$$

$$K = 46355.91 \text{ lb/ft}$$

(d) In weight of the tank = $W = ?$

$$\omega n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K \times g}{W}}$$

$$\omega n^2 = K \times g / W$$

$$\Rightarrow W = K \times g / \omega n^2$$

also $\omega n = 2\pi / T_n$

$$W = K \times g / \left(\frac{4\pi^2}{T_n^2} \right) = K \times g \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{46355.91 \times 32.2 \frac{\text{ft}}{\text{sec}} \left(\frac{\text{lb}}{\text{ft}} \right)}{4\pi^2}$$

$$W = 46355.91 \times 32.2 \times \left(\frac{(0.46)^2}{4(3.14)^2} \right)$$

$$W = 8008.61 \text{ lb}$$

$$\Rightarrow W = 8.008 \text{ K}$$

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(e) Dumping Coefficient: $\dot{C} = ?$

we know that:

$$\delta = \frac{C}{2m\omega_n}$$

$$\Rightarrow C = \delta (2m\omega_n) = \delta (2m \frac{2\pi}{T_n})$$

put values

$$C = \frac{0.462 (2 (\frac{8008.61}{32.2}))}{0.46 \text{ sec}}$$

$$C = \boxed{49.959 \text{ lb sec/ft}}$$

P.T.O.

(f) No of cycles to reduce the displacement amplitude to 0.5

$$j = ?$$

$$j = \frac{1}{2\pi\zeta} \ln \left(\frac{4.156}{a_{j+1}} \right)$$

$$\frac{1}{2(3.14)(0.0482)} \ln \left(\frac{7.766}{0.5} \right) = 9.1$$

So $\boxed{j = 9 \text{ cycles.}}$

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Given Data:

- * Damping ratio of reinforce concrete with considerable cladding = 3-5%
- * So we take $\gamma = 3\%$
- * other data are taken from question No:1

Required value:

Develop and solve the eqn of motion for vibration at free end=?;

Develop an equation showing variation in equivalent state forces with time=?

Sol:

As we know that EOM (equation of motion) for damped free vibration is;

$$Ku + cu + mu = 0 \quad \text{--- (1)}$$

As we know that from question no. 1 data

$$K = 90625 \text{ lb/ft}$$

$$m = 240.46 \text{ rad/sec}$$

$$\omega_n = 19.983$$

As we know that,

$$c = \zeta \times 2m\omega_n$$

$$c = 0.03 \times 2 \times 240.46 \times 19.98$$

$$c = 288.263 \text{ lb}^{\text{sec}}/\text{ft}$$

P.T.O

By putting value in eq (1) we get.

$$90625u + 288.263u + 240.4u = 0$$

Solution to the EOM for damped free vibration is,

$$u(t) = e^{-\gamma \omega_n t} \left[4(u_0) \cos(\omega_n t) + \frac{1}{\omega_D} \left[\dot{u}(0) + u(0) \gamma \omega_n \right] \sin \omega_n t \right]$$

Here;

$$\omega_D = 19.984 \text{ rad/sec}$$

$$u(t) = e^{-(0.03 \times 19.984 \times t)} \left[\frac{1}{24} \times \cos(19.984t) \right]$$

$$+ \frac{1}{19.984} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.984 \times \sin(19.984t) \right]$$

P.T.O.

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$$u(t) = e^{-0.599t} \left[0.0416 \cos(19.984t) + 0.0247 \sin(19.984t) \right]$$

we know that:

$$f_s(t) = k \cdot u(t)$$

$$f_s(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.599t} \left[3806.25 \cos(19.984t) + 2175 \sin(19.983t) \right]$$

→ X → Ans

Damped Free vibration.

