Course: Calculus and analytical geometry
Program: BS (SE, CS)
Instructor: Muhammad Abrar Khan
Examination: Midterm Assignment
Total Marks: 30
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Note: Attempt all questions. Use examples and diagrams where necessary.

Q.1

a) Differentiate $\frac{2x^3-3x^2+5}{x^2+1}$ with respect to x.

b) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ with respect to x.

Q.2

a) Find $\frac{dy}{dx}$ if y = $(1+2\sqrt{x})^3.x^{2/3}$ using chain rule.

b) Find $\frac{dy}{dx}$ if y = $\sqrt{\frac{1-x}{1+x}}$ using chain rule.

Q.3

a) Find the Integration of $\int \frac{1}{\sqrt{x^3}}$ dx.

b) Find the Integration of $\int \frac{1}{(6x+7)^6}$dx.

Q No 1 Part(a) :-

Differentiate $\frac{2x^3-3x^2+5}{x^2+1}$ with respect to x.

$$= \frac{2x^3-3x^2+5}{x^2+1}$$

By Quotient Rule

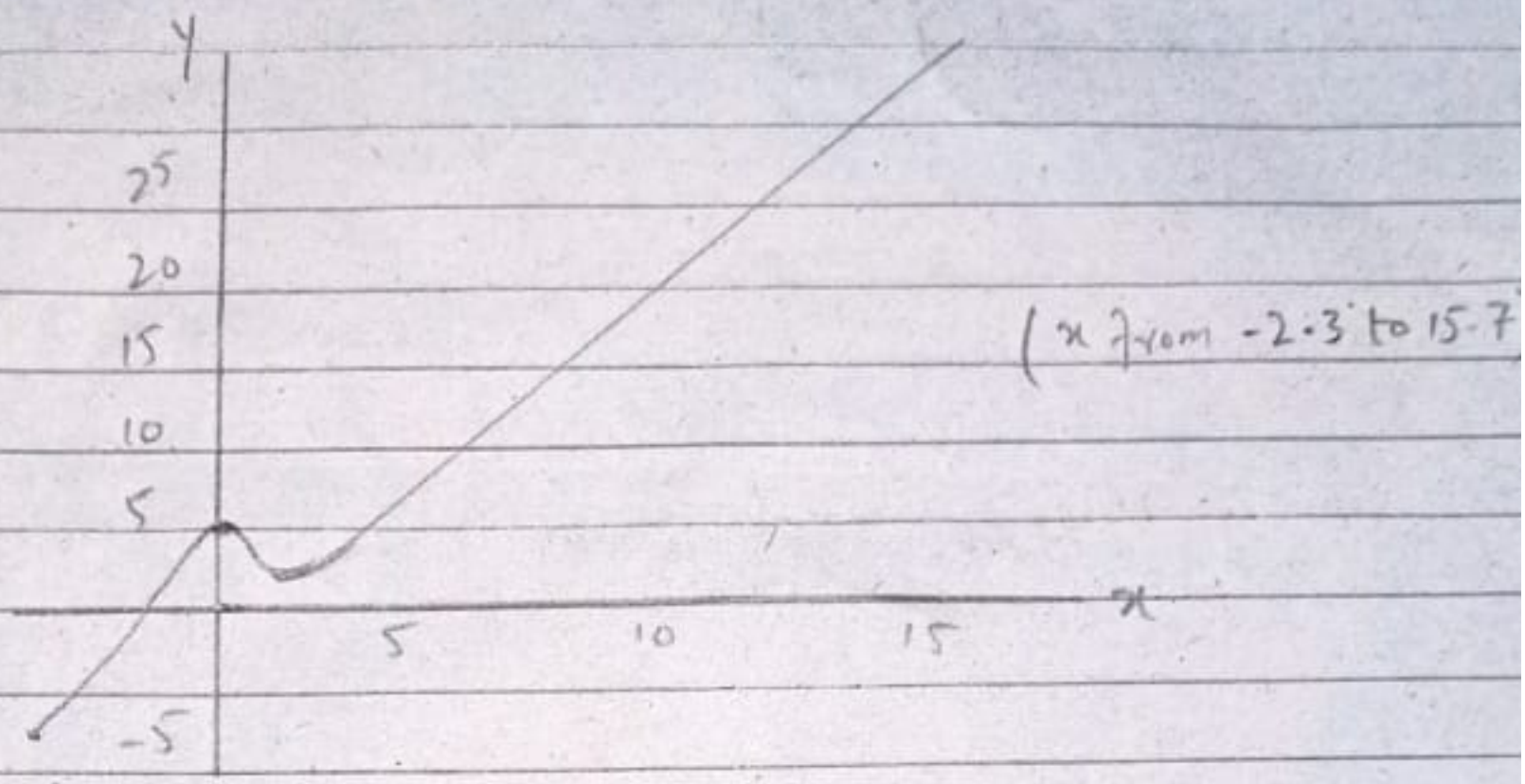
$$\frac{d}{dx} = \frac{(x^2+1)\frac{d}{dx}(2x^3-3x^2+5)-(2x^3-3x^2+5)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(6x^2-6x)-(2x^3-3x^2+5)(2x)}{(x^2+1)^2}$$

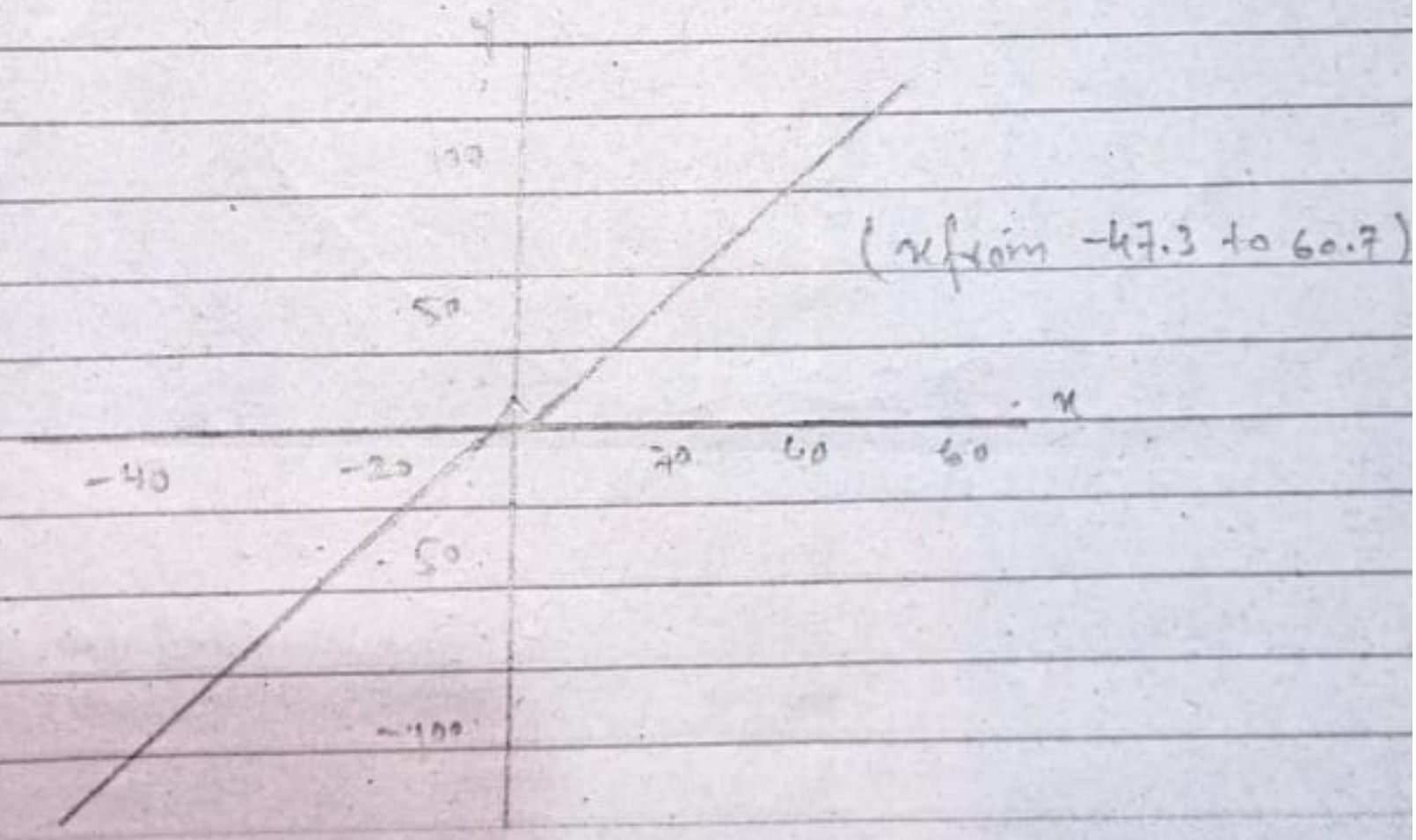
$$= \frac{6x^4-6x^3+6x^2-6x-4x^4+6x^3-10x}{(x^2+1)^2}$$

$$= \frac{2x^4-6x^3-16x}{(x^2+1)^2}$$ Ans.

Q1 Part (a)



(x from -2.3 to 15.7)



(x from -47.3 to 60.7)

Q No 1 Part (b)

Differentiate $\frac{(x^2+1)^2}{x^2-1}$ with respect to x.

$$\frac{(x^2+1)^2}{x^2-1}$$

By Quotient Rule.

$$\frac{d}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\,2(x^2+1)(2x) - (x^2+1)^2(2x)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)(x^2+1)\,4x - 2x(x^2+1)^2}{(x^2-1)^2}$$

$$= \frac{4x[x^4+x^3-x^2-1] - 2x(x^4+1+2x^2)}{(x^2-1)^2}$$

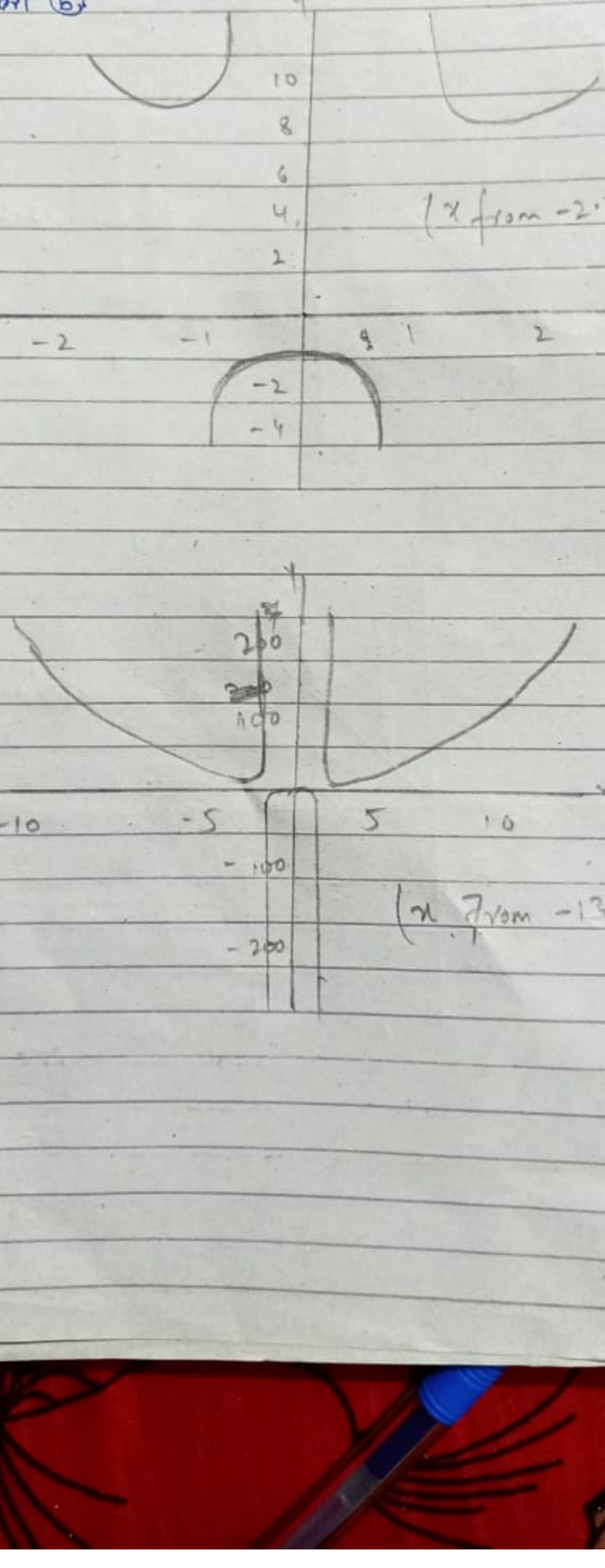
$$= \frac{4x[x^4-1] - 2x^5 - 2x - 4x^3}{(x^2-1)^2}$$

$$= \frac{4x^5 - 4x - 2x^5 - 2x - 4x^3}{(x^2-1)^2}$$

$$= \frac{2x^5 - 6x - 4x^3}{(x^2-1)^2}$$

$$= \frac{2x^5 - 4x^3 - 6x}{(x^2-1)^2}$$ Ans.

Q1 Part (b)



(x from -2.3 to 2.3)

(x from -13.8 to 13.8)

Q No 2 Part (a) :-

Find $\frac{dy}{dx}$ if $y = (1+2\sqrt{x})^3 \cdot x^{2/3}$ using chain rule.

$y = (1+2\sqrt{x})^3 \cdot x^{2/3}$

By Chain Rule $\Rightarrow \frac{dy}{dx} \cdot \frac{dy}{du} \cdot \frac{du}{dx}$

$\frac{dy}{dx} = (1+2\sqrt{x})^3 (x^2)^{2/3}$

$\frac{dy}{dx} = (1+2\sqrt{x})^3 \, 3\sqrt{x^2}$

$\frac{dy}{dx} = (x^3 + 2x^2\sqrt{x})^{3+1/3}$

$\frac{dy}{dx} = (x^2 + 2x^2 (x)^{1/2})^{10/3}$

$\frac{dy}{dx} = (x^2 + 2x^2\sqrt{x})^{10/3}$

$(x^2 + 2x^2\sqrt{x})^{10/3}$

$U = x^2 + 2x^2\sqrt{x}$

(5)

$$\frac{dy}{dx} = 2x + \sqrt{2x^2}\,1/2\,x + \sqrt{x}\,4x)$$

$$dy = 2x + \frac{\sqrt{2}x^3}{\sqrt{2}} + 4x \cdot x^{1/2}$$

$$dy = 2x + x^3 + 4x^{1+1/2}$$

$$\frac{dy}{dx} = 2x + x^3 + 4^{3/2}$$

$$\frac{dy}{du} = u^{10/3}$$

$$\frac{dy}{du} = \frac{d}{du}\, u^{10/3}$$

$$\frac{dy}{du} = 10^{7/3}$$

$$\frac{dy}{dx} = 10^{7/3} \cdot 2x + x^3 + 4^{3/2}$$

$$\frac{dy}{dx} = 10^{7/3} \cdot 2x + x^3 + 4^{3/2} \quad \text{Ans.}$$

Q2 Part (b)

Differentiate

Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1-x}{1+x}}$ using chain rule.

$$y = \sqrt{\frac{1-x}{1+x}}$$

By Chain Rule.

$$\frac{dy}{dx} \cdot \frac{dy}{du} \cdot \frac{du}{dx}$$

let $du \; v = \frac{1-x}{dx} \cdot \frac{}{1+x}$

$$= \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$= \frac{-(-x-1+x}{(1+x)^2}$$

$$\frac{du}{dx} = \frac{-2}{(1+x)^2}$$

(7)

$$y = \sqrt{u}$$

$$y = u^{1/2}$$

$$\frac{dy}{du} = \frac{d}{du} u^{1/2}$$

$$\frac{dy}{du} = \frac{1}{2} u^{\frac{-1}{2}}$$

$$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$$

Putting Values

$$\frac{dy}{dx} = \frac{1}{2\sqrt{u}} \cdot \frac{-2}{(1+x)^2}$$

$$\frac{dy}{dx} = \left(\frac{1}{2\sqrt{\frac{1-x}{1+x}}}\right)\left(\frac{-2}{(1+x)^2}\right)$$

$$\frac{dy}{dx} = \frac{1}{x} \frac{(1+x)^{1/2}}{(1-x)^{1/2}} \frac{-x}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{(1-x)^{1/2}(1+x)^{2-1/2}}$$

$$\frac{dy}{dx} = \frac{-1}{(1-x)^{1/2}(1+x)^{3/2}}.$$ Ans.

Q No 3 Part (a) :-

Find the Integration of $\int \frac{1}{\sqrt{x^3}} dx$

$$= \int \frac{1}{\sqrt{x^3}} dx$$

$$= \int \frac{1}{x^{3/2}} dx$$

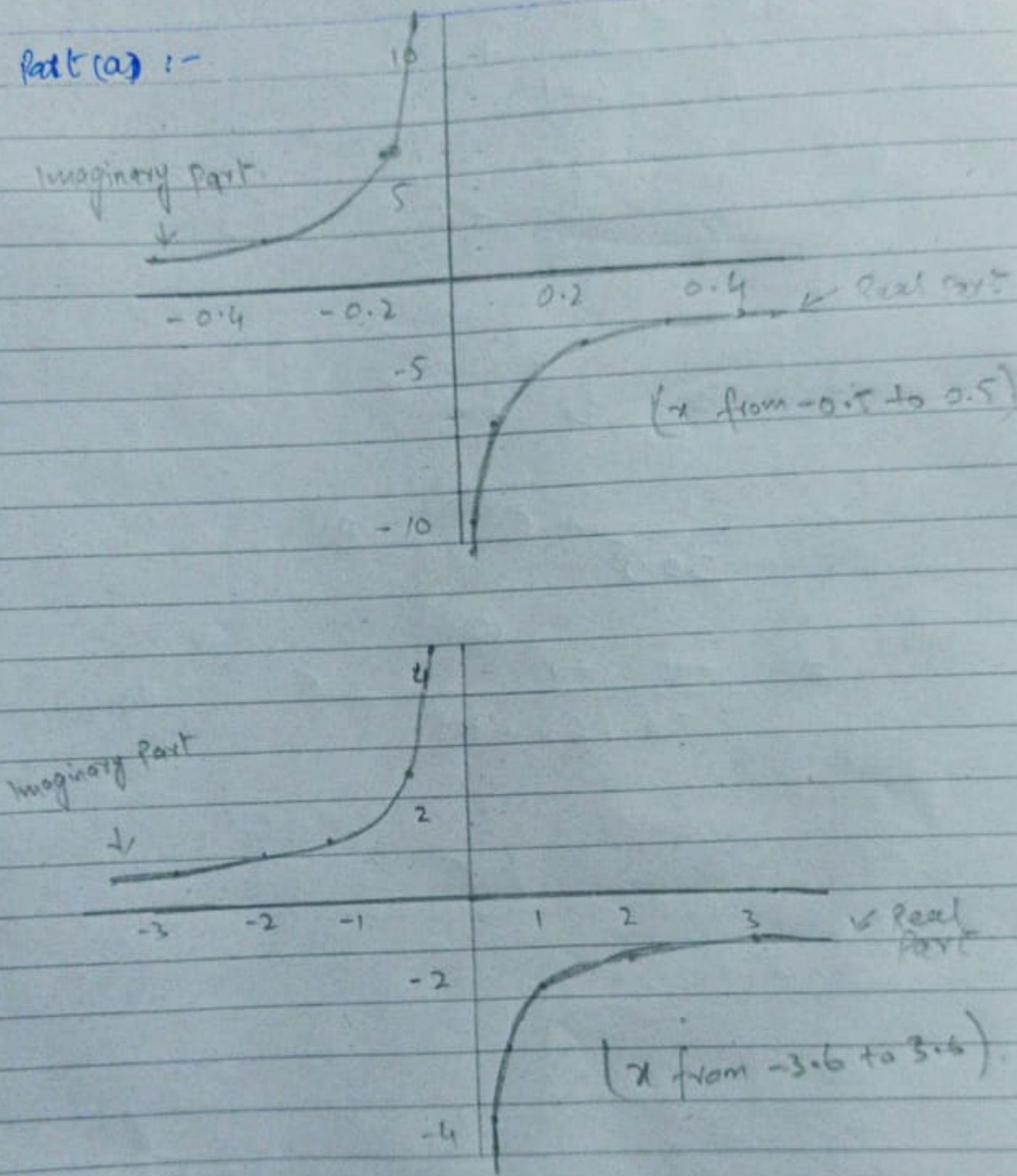
$$= \int x^{-3/2} dx$$

$$= \frac{x^{\frac{-3}{2}+1}}{\frac{-3}{2}+1} + C$$

$$= \frac{x^{-1/2}}{-1/2} + C$$

$$= \frac{-2}{\sqrt{x}} + C \quad \text{Ans.}$$

Q3 Part (a) :-


Imaginary Part
10
5
-0.4
-0.2
0.2
0.4
Real Part
-5
-10
(x from -0.5 to 0.5)
Imaginary Part
4
2
-3
-2
-1
1
2
3
Real Part
-2
-4
(x from -3.6 to 3.6)

Q No 3 Part (b).

Find the integration of $\int \frac{1}{(6x+7)^6} dx$

$$\int \frac{1}{(6x+7)^6} dx$$

$$= \int (6x+7)^{-6} dx$$

Let $u = 6x + 7$

$$du = 6\, dx$$

$$\frac{du}{6} = dx$$

$$= \int u^{-6} \frac{du}{6}$$

$$= \frac{1}{6} \int u^{-6} du$$

$$= \frac{1}{6} \frac{u^{-6+1}}{-6+1} + C$$

$$= \frac{1}{6} \frac{u^{-5}}{-5} + C$$

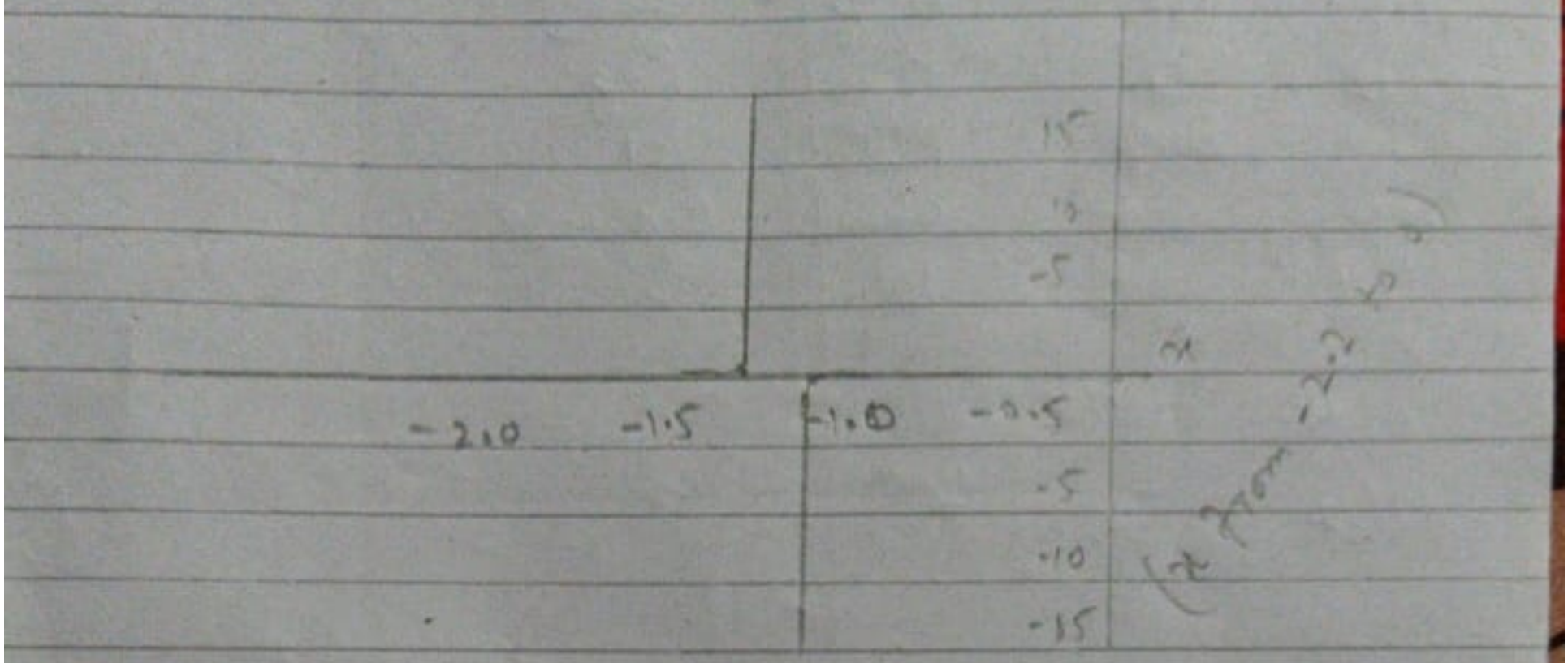
$$= -\frac{1}{30} u^{-5} + C$$

$$= \frac{-1}{30u^5} + C \quad \text{Ans.}$$

OR

$$= \frac{-1}{30(6x+7)^5} + C \quad \text{Ans}$$

Q3 Part b



(x from -7.3 to 5)