Name = Asfandyar

I.D = 7274

Assignment ==> Differential Equation

Submitted To :-> Mam Shomaila Mazhar

Cauchy Euler Method

Q NO :-> (01)

$$x^3y''' + 2x^2y^1 + 2y = 10x + \frac{10}{x}$$

Solution

Put $x = e^t$ then

$$\frac{dx}{dx} = e^t \Rightarrow \frac{dt}{dt} = e^{-t}$$

Now

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot e^{-t}$$

OR

$$y' = \frac{dy}{dx} = e^{-t} Dy \quad \because \frac{d}{dt} \Rightarrow D$$

P.T.O

Similarly $y'' = e^{-2t}[D(D-1)]y$

$y''' = e^{-3t}[D(D-1)D-2)y$

Using these Values in ①

$e^{3t} e^{-3t} [D(D-1)(D-2)y + 2e^{2t} \cdot e^{-2t}[D(D-1)]y$

$+ 2y = 10e^{t} + 10^{-et}$

$\Rightarrow (D^3 - 3D^2 + 2D)y + (2D^2 - 2D)y + 2y = 10e^{t} + 10e^{-t}$

$\Rightarrow D^3 y - D^2 y + 2y = 10e^{t} + 10e^{-t}$

$\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + 2y = 10e^{t} + 10e^{-t} \longrightarrow ②$

The assoisaled homogenious equation of ②

$\frac{d^3y}{dt^3} - \frac{d^3y}{dt^3} + 2y = 0$

Say, $\frac{d}{dt^2} = K^2, \frac{d^3}{dy^3} = K^3$

$\Rightarrow (K^3y - K^2y + 2y) = 0$

$\Rightarrow (K^3 - K^3 + 2)y = 0$

For non - trivial Sol-, $y \neq 0 \Rightarrow$

$K^3 - K^2 + 2 = 0$

P.T.O

=> Roots are

$$k = -1 \pm 2$$

=> $y_c(t) = A e^{-t} + (B \cos t + C \sin t) e^{t}$

When is complimentry sol.

Q No :-> (03)

$$x^2 y'' + 2xy' - by = 10x^2 = y(1) = 1$$
$$y'(1) = -6$$

Solution

$$x^2 y'' + 2xy' = by = 10x^2 \rightarrow ①$$

Let

$x = e^t$ i.e $t = \log x$ $\quad y(1) = 1$

$y'(1) = -6$

Now

$$xy' = \Delta y \Rightarrow x^2 y' = \Delta(\Delta - 1) y$$

where $\Delta = \frac{d}{dt}$.

Then equation ① => $[\Delta(\Delta-1) + 2\Delta - 6] y = 10e^{2t}$

$$[\Delta^2 \Delta + 2\Delta - 6] y = 10e^{2t}$$

P.T.O

$\Rightarrow [\Delta^2 + \Delta - 6]y = 10e^{2t}$

Char term Equation $\Delta^2 + \Delta - 6 = 0$

$\Delta + 3\Delta - 2\Delta - 6 = 0$

$\Delta = -3, \Delta = 2$

Complementry function

C.F $C_1 e^{-3t} + C_2 e^{2t}$

Also P. Integral

$P.I = \frac{1}{\Delta^2 + \Delta - 6} 10e^{2t}$

$= 10 \frac{1}{(2)^2 + 2 - 6} e^{2t}$ replace Δ by 2

case of failure

$P.I = 10 \cdot t \frac{1}{2\Delta + 1} e^{2t} = 10t \frac{1}{2(2) + 1} e^{2t}$

$= 10t \frac{1}{5} e^{2t} = 2te^{2t}$

Hence general sol. $y = C.f + P.I$

$y = C_1 e^{-3t} + C_2 e^{2t} e^{2t}$

$C_1 x^3 + C_2 x^2 + 2 (\log x) x^2$

P.T.O

Apply Initial Cundition

$y(1) = 1$ we get
$1 = C_1 + C_2 + 0 \rightarrow$ A
a $y(1) = -6$

$y' = 3C_1 x^{-4} + 2C_2 + x 2x^4 x \log x$

$-6 = 3C_1 + 2C_2 + 2 + 0$

$-3C_1 + 2C_2 = -8 \rightarrow$ B

eq A $\times 3$ and add with B

$3 = 3\cancel{C_1} + 3C_2$
$-8 = -3\cancel{C_1} + 2C_2$

$5C_2 = -5$
$\boxed{C_2 = -1}$

eq A $\Rightarrow 1 = C_1 - 1$
$\boxed{C_1 = 2}$

Thus A $\Rightarrow$ $\boxed{y = 3x^{-3} - x^2 + 2x^2 \log x}$

P.T.O

Page :→ (06)

Q No: → (04)

$x^2y'' + 7xy' + 5y = x^5$;

$y(0) = 2$ & $y'(1) = 2$

solution

Let

$x = e^t \Rightarrow t = \log x, \Delta \frac{d}{dt}$

Now $xy' = \Delta y' \Rightarrow x^2y' = \Delta(\Delta-1)y$

$(\Delta(\Delta-1) + 7\Delta + 5)y = e^{5t}$

$(\Delta^2 - \Delta + 7\Delta + 5)y = e^{5t}$

$(\Delta^2 + 6\Delta + 5)y = e^{5t}$

Char eq is $\Delta^2 + 6\Delta + 5 = 0$

$\Delta^2 + 5\Delta + \Delta + 5 = 0$

$\Delta = 5, -1$

Complementare eq is

$C.F = C_1e^{5t} + C_2e^{-t}$

P.T.O

$\Rightarrow$ P. Integral

$$P.I = \frac{1}{\Delta^2 + 6\Delta + 5} e^{5t}$$

$$= \frac{1}{5^2 + 6(5) + 5} \quad \text{replacing } \Delta \text{ by } 5$$

$$= \frac{1}{60} e^{5t}$$

Thus

$$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{5t}$$

$$y = C_1 x^{-5} + C_2 x^{-2} + \frac{1}{60} x^5$$

$$y = 5C_1 x^{-6} - C_2 x^{-2} + \frac{1}{12} x^4$$

$y(0) = 2 \quad x = 0, y = 2$

$$2 = C_1 + C_2 + \frac{1}{60}$$

$$C_1 + C_2 = 119/60 \rightarrow (A)$$

$y'(1) = 2 \quad x = 1, y' = 2$

$$2 = 5C_1 - C_2 + 1/12$$

$$5C_1 - C_2 = 23/12 \longrightarrow (B)$$

$$A + B - 4C_1 = \frac{234}{60} \Rightarrow 4 = \frac{-117}{120}$$

P.T.O

Now

$$y = \frac{-117}{120} x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$$

$C_1 = \frac{-117}{120}$ put in Ⓐ $-\frac{117}{120} + C_2 - \frac{119}{60}$

$$C_2 = \frac{119}{60} + \frac{117}{120}$$

$$\boxed{= \frac{238 + 117}{120} = \frac{355}{120}}$$

QNo :-> (05)

$$(x+1)^2 y'' - 3(x+1) y' + 4y = x^2$$

Solution

$$(x+1)^2 y' - 3(x+1) y' + 4y = x^2 \longrightarrow ①$$

Let

$$x + 1 = e^t \Rightarrow x = e^t - 1$$

Diff $\log(x+1) = t$

Also $(x+1) y' = Dy$, $\left\{ \frac{d}{dt} = \Delta \right.$ and $\left. D = \frac{d}{dx} \right\}$

P.T.O

Then eq ① $\Rightarrow (\Delta(\Delta-1)-3\Delta+4)y=(e^t-1)^2$

$(\Delta^2-4\Delta+4)y=e^{2t}-2e^t+1$

Char eq is $\Delta^2-4\Delta+4=0$

$(\Delta-2)^2=0$

$\Delta=2,2$

This complementry function is

$C.F=(C_1+C_2t)e^{2t}$

Also particular Integral is

$$P.I=\frac{1}{(\Delta-2)^2}(e^{2t}-2e^t+1)$$

$$=\frac{1}{(\Delta-2)^2}(e^t-2\frac{1}{(\Delta-2)^2}e^t\frac{1}{(\Delta-2)^2}\rightarrow ②$$

Now

$$\frac{1}{(\Delta-2)^2}e^{2t}=\frac{1}{(2-2)^2}e^{2t}=\frac{1}{0}e^{2t}$$

case of failure

$$\frac{1}{(\Delta-2)^2}e^{2t}=t\frac{1}{2(1-2)^2}e^t=\frac{t^2e^t}{2}$$

and $2\frac{1}{\Delta-2^2}e^t=2\frac{1}{(1-2)^2}e^t=2e^t$

P.T.O

and $\frac{1}{(\Delta-2)^2}(1) = \frac{1}{(\Delta-2)^2} e^{0t} = \frac{1}{4}$

eq (2) $\Rightarrow$ P.I $\frac{1}{2}t^2 e^{st} - 2e^t + \frac{1}{4}$

Hence Complete Solution is

$y = C.F + P.I$

$y = (C_1 + C_2 t)e^{2t} + \frac{1}{2}t^2 e^{2t} - 2e^t + \frac{1}{4}$

repect Value of e^t

$y = (C_1 + C_2 \log(x+1)) (x+1)^2 + \frac{1}{2}[\log(x+1)^2 (x+1)^2] - 2(x+1) + \frac{1}{4}$

OR

$y = C_1 + C_2 \log(x+1) \cdot (x+1)^2 + \frac{1}{2}(\log(x+1)^2 (x+1)] - 2x - 7/4$

Which is the required

P.T.O

QNO :→ (02)

$$x^3y''' + 4x^2y'' - 5xy' - 15y = x^4$$

Solution

$$x^3y''' + 4x^2y'' - 5xy' - 15y = x^4 \rightarrow (1)$$

let

$xy = e^t$ of $x = e^t - 1$

Diff $\log(x-y) = t$

Also

$(xy) = \Delta y$ $\left\{ \frac{d}{dt} = \Delta \text{ and } \Delta = \frac{d}{dx} \right\}$

Then eq (1) $(\Delta(\Delta-1) - 15\Delta + 5)y = (e^t - 1)$

$(\Delta^2 - 15\Delta + 5)y = e^{2t} - 2e^t + 1$

Char eq is $(\Delta^2 - 15\Delta + 5)y = 0$

$(\Delta - 5)^2 = 0$

$\Delta = 5, 5$

The Complemtry funtion is

$CF = (C_1 + C_2 t)e^{2t}$

Also ~~pro~~ particular Integral is

$PI = 1/(\Delta-5)^2 (e^{2t} - 2e^t + 1) \rightarrow (2)$

$= \frac{1}{(\Delta-2)^2} e^{2t} - 5 1/(\Delta-5)^2 e^t + 1/(\Delta-5)^2$

Now $1/(\Delta-5)^2 e^{2t} = 1/(5-5)^2 e^{2t} = 1/0 \, e^{2t}$

&

case of feiture

$$\frac{1}{(\Delta-5)^2} e^{2t} = t \frac{1}{5(1-3)^2} e_1 = \frac{t^2 e^t}{5}$$

and $5 \frac{1}{(\Delta-5)^2} e^t = 5 \frac{1}{(1-5)^2} et = \frac{t^2 e^6}{5}$

and $\frac{1}{(\Delta-5)^2} e^t = 5 \frac{1}{(1-5)^2} e^t - 5e^t$

and $\frac{1}{(\Delta-5)^2} (1) = \frac{1}{(\Delta-5)^2} e^t = \frac{1}{15}$

eq ② $\longrightarrow PI = \frac{1}{5} t^2 e^{2t} - 5e^t + \frac{1}{15}$

Hence complete solution is

$y = C.f + PI$

$y = (c_1 + c_2 t) e^{2t} + \frac{1}{5} t^2 e^{2t} - 5et \frac{1}{15}$

$y = Cf + PI$

$y = C_1 + C_2 t) e^{2t} + \frac{1}{5} t^2 e^{2t} - 5e^t \frac{1}{15}$

Refect value e^t

$y = C_1 + C_2 \log (x+1) (x+1)^2 + \frac{1}{2}$

$[(\log (x+1)^2 (x+1)^2] - 5(x+1)$

$+ \frac{1}{15}$

CW 110