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Question#01:

A prototype gate valve which control the flow in a pipe system conveying paraffin is to be studied in a model. List the significant variables on which the pressure drop across the valve would depend. Perform dimensional analysis to obtain the relevant non-dimensional groups.

A $1/5$ scale model is built to determine the pressure drop across the valve with water as the working fluid.

- For a particular opening, when the velocity of paraffin in the prototype is 3.0 m s^{-1} . what should be the velocity of water in the model for dynamic similarity?
- what is the ratio of the quantities of flow in prototype and model?
- Find the pressure drop in prototype if is 60 kPa in the model?

The density and viscosity of paraffin are 800 kg m^{-3} and $0.009 \text{ kg m}^{-1} \text{ s}^{-1}$ respectively. Take kinematic viscosity of water as $1.0 \times 10^{-6} \text{ m}^2/\text{s}$.

The pressure drop ΔP is expected to depend upon the gate opening h , overall depth d , the velocity V , density ρ and viscosity μ .

Solution:

First of all, we have to find all the relevant variables and its dimensions.

$$1) \Delta P \Rightarrow ML^{-1}T^{-2}$$

$$2) h \Rightarrow L$$

$$3) d \Rightarrow L$$

$$4) V \Rightarrow LT^{-1}$$

$$5) P \Rightarrow ML^{-3}$$

$$6) \mu \Rightarrow ML^{-1}T^{-1}$$

So,

the number of total variables = 6 = n

number of independent variables = m = 3 [M L T]

number of non-dimensional groups = n - m

$$= 6 - 3$$

$$= 3$$

Choose m = 3 Scaling variables

↳ geometric (d)

↳ Kinematic / time dependent (V)

↳ dynamic / mass dependent (P)

Now we have to form dimensionless groups by non-dimensionalising the remaining variables which are ΔP , h and μ .

$$\hookrightarrow \pi_1 = \Delta p d^a v^b \rho^c$$

$$\begin{aligned} M^0 L^0 T^0 &= (ML^{-1}T^{-2})(L)^a (LT^{-1})^b (ML^{-3})^c \\ &= M^{1+c} \cdot L^{-1+a+b-3c} \cdot T^{-2-b} \end{aligned}$$

$$\Rightarrow M \Rightarrow 0 = 1+c \Rightarrow c = -1$$

$$\Rightarrow L \Rightarrow 0 = -1+a+b-3c \Rightarrow a = 1+3c-b = 0$$

$$\Rightarrow T \Rightarrow 0 = -2-b \Rightarrow b = -2$$

$$\begin{aligned} \pi_1 &= \Delta p d^0 v^{-2} \rho^{-1} \\ &= \Delta p v^{-2} \rho^{-1} \end{aligned}$$

$$\pi_1 = \frac{\Delta p}{\rho v^2}$$

$$\begin{aligned} a &= 1+3c-b \\ a &= 1+3(-1)-(-2) \\ a &= 1-3+2 \\ a &= 0 \end{aligned}$$

$$\hookrightarrow \pi_2 = \frac{h}{d} \rightarrow h \text{ is a length}$$

$$\hookrightarrow \pi_3 = \mu d^a v^b \rho^c$$

$$\begin{aligned} \Rightarrow M^0 L^0 T^0 &= (ML^{-1}T^{-1})(L)^a (LT^{-1})^b (ML^{-3})^c \\ &= M^{1+c} \cdot L^{-1+a+b-3c} \cdot T^{-1-b} \end{aligned}$$

$$\Rightarrow M \Rightarrow 0 = 1+c \Rightarrow c = -1$$

$$\Rightarrow L \Rightarrow 0 = -1+a+b-3c \Rightarrow a = 1-b+3c$$

$$\Rightarrow T \Rightarrow 0 = -1-b \Rightarrow b = -1 \quad \left| \begin{aligned} a &= 1-(-1)+3(-1) \\ a &= 1+1-3 \\ a &= 2-3 \\ a &= -1 \end{aligned} \right.$$

$$\Rightarrow \pi_3 = \mu d^{-1} v^{-1} \rho^{-1}$$
$$= \frac{\mu}{\rho v d}$$

$$\Rightarrow \pi'_3 = (\pi_3)^{-1} = \frac{\rho v d}{\mu}$$

Hence dimensional analysis yields

$$\pi_1 = f(\pi_2, \pi'_3)$$

$$\Rightarrow \frac{\Delta P}{\rho v^2} = f\left(\frac{h}{d}, \frac{\rho v d}{\mu}\right)$$

a) As for the dynamic similarities, all non-dimensional groups must be same in model as well as prototype so,

$$\pi_1 = \left[\frac{\Delta P}{\rho v^2} \right]_p = \left[\frac{\Delta P}{\rho v^2} \right]_m$$

$$\pi_2 = \left[\frac{h}{d} \right]_p = \left[\frac{h}{d} \right]_m$$

$$\pi_3 = \left[\frac{\rho v d}{\mu} \right]_p = \left[\frac{\rho v d}{\mu} \right]_m$$

From π_3 , we have the following velocity ratio

$$\frac{V_p}{V_m} = \frac{(\mu/\rho)_p}{(\mu/\rho)_m} \cdot \frac{d_m}{d_p}$$
$$= \frac{0.002/800}{1.0 \times 10^{-6}} \times \frac{1}{5}$$

$$\frac{V_p}{V_m} = 0.5$$

Hence

$$V_m = \frac{V_p}{0.5} = \frac{3.0}{0.5}$$

$$V_m = 6 \text{ m/s}$$

b) The ratio of quantities of flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{velocity} \times \text{area})_p}{(\text{velocity} \times \text{area})_m}$$

$$= \frac{V_p}{V_m} \cdot \left[\frac{d_p}{d_m} \right]^3$$

$$= 0.5 \times 5^3$$

$$\Rightarrow \frac{Q_p}{Q_m} = 12.5$$

c) Now the pressure drop in the prototype

$$\text{As } \pi_1 = \left[\frac{\Delta P}{\rho V^3} \right]_p = \left[\frac{\Delta P}{\rho V^3} \right]_m$$

$$\Rightarrow \frac{(\Delta P)_p}{(\Delta P)_m} = \frac{\rho_p}{\rho_m} \cdot \left(\frac{V_p}{V_m} \right)^3$$

$$= \frac{800}{1000} \times (0.5)^3$$

$$\frac{(\Delta P)_p}{(\Delta P)_m} = 0.2$$

$$\begin{aligned} \Rightarrow (\Delta P)_p &= 0.2 \times \Delta P_m \\ &= 0.2 \times 60 \end{aligned}$$

$$\boxed{\Delta P_p = 12 \text{ kPa}}$$

Question #09

Design a practical profile of gravity dam with the following data

- 1) Maximum depth of water in Reservoir
(First two digits of $R = 77$)
- 2) Specific gravity of dam material is G
- 3) Allowable compressive strength of material for dam masonry is T/m^2
- 4) Height of wave is H_w

Given data:

$$R = 7799$$

Maximum depth of water in Reservoir,
 $= H_w = 77 \text{ m}$

Specific gravity of dam material $= G = G$
 let suppose $G = 2.1$

Allowable compressive stress for the Dam
 masonry $= \sigma_{ay} = 779 \text{ T/m}^2$

Height of wave $= h_w = 1.4 \text{ m}$

$$M = 0.7 \text{ m}$$

$$C_u = 0$$

Solution:Step#01:

$$H_{\text{limiting}} = \frac{C_{au}}{\gamma_w (G - C_u + 1)}$$

$$= \frac{779 \times 1000}{1000 (2.1 - 0 + 1)}$$

$$\Rightarrow H_{\text{limiting}} = 251.290 \text{ m} > 77 \text{ m}$$

So it is a low gravity Dam.

Step#02:Top width "a"

$$\text{Free board} = 1.5 \times h_w$$

$$= 1.5 \times 1.4$$

$$\boxed{F.B = 2.1 \text{ m}}$$

$$\text{Height of dam} = H_D = H_w + F.B$$

$$= 77 + 2.1$$

$$\Rightarrow \boxed{H_D = 79.1 \text{ m}}$$

As

$$a = 14\% \text{ of } H_D$$

$$= \frac{14}{100} \times 79.1$$

$$\Rightarrow \boxed{a = 11.074 \text{ m}}$$

Step#03:-Base width "b" (without offset)

i) For no sliding criteria

$$b' = \frac{Hw}{\mu G}$$

$$= \frac{77}{0.7 \times 2.1}$$

$$b' = 52.381 \text{ m} \approx 52 \text{ m}$$

ii) For no tension criteria

$$b' = \frac{Hw}{\sqrt{G}}$$

$$= \frac{77}{\sqrt{2.1}}$$

$$b' = 53 \text{ m}$$

we will use $b' = 53 \text{ m}$ Step#04:Depth of vertical portion on u/s side

$$h' = 2a\sqrt{G - c_u}$$

$$= 2 \times 11.074 \sqrt{2.1 - 0}$$

$$\Rightarrow h' = 32.095 \text{ m}$$

Step #05

$$\begin{aligned} \text{Upstream offset} &= \frac{a}{16} \\ &= \frac{11.074}{16} \\ &= 0.692 \text{ m} \end{aligned}$$

Step #06

Depth below water level to the end of inclined portion in U/s $= 3.14 a \sqrt{g}$

$$\begin{aligned} &= 3.14 \times 11.074 \times \sqrt{2.1} \\ &= 50.39 \text{ m} \end{aligned}$$

Step #07:

Total width of base of the Dam

$$b = b' + \frac{a}{16} = 53 + 0.692 = 53.692 \text{ m}$$

Step #08:

$$\tan \theta = \frac{b'}{H} = \frac{53}{77}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{53}{77} \right) = 34.54^\circ$$

Step #09

Depth of vertical portion on D/s
(from WL on U/s side)

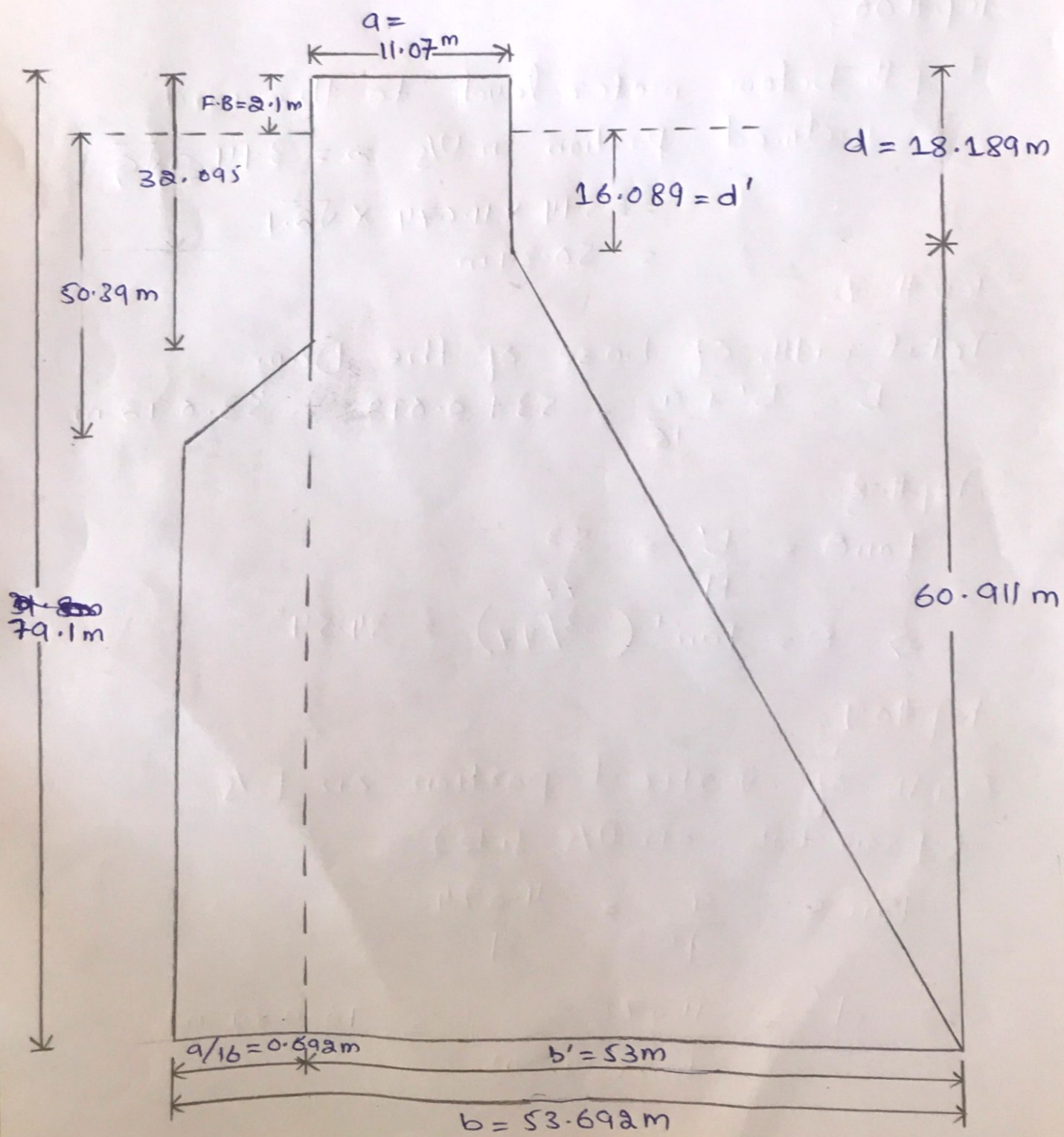
$$\tan \theta = \frac{a}{d'} = \frac{11.074}{d}$$

$$\Rightarrow d' = \frac{11.074}{\tan(34.54)} = 16.089 \text{ m}$$

Depth of vertical portion

$$d = d' + F.B$$
$$= 16.089 + 2.1$$

$$d = 18.189m$$



Question #04Answer:Factors affecting fall velocity-

- 1) Particle diameter.
- 2) Particle density.
- 3) Particle concentration.
- 4) Particle shape.
- 5) viscosity of water.
- 6) Turbulence of water.

1) Particle diameter:

Sediment particle diameter is directly proportional to the fall velocity. It is because if the size of the particle is greater, it will tend to move faster as compared to the particle of smaller size and thus there will be more gravitational force on particle of greater size so it will fall quickly due to its heavy weight.

2) Particle density:

Particle density is also directly proportional to the fall velocity as the particles having high density tends to settle down quickly as compared to particle with low density.

3) Particle concentration:-

concentration of particle size will considerably effect its fall velocity as the section having greater concentration will settle down at the place thus causing more fall of velocity comparing with section of low concentration.

4) Particle Shape:

Sediment particle shape also effects on fall velocity. Non spherical analogue particle falls up 75% slower than equivalent sphere shape particle.

Particle having regular shape tends to effect fall velocity more than irregular shape particle.

viscosity
5) velocity of water:

Fluid velocity through porous medium is approximately inverse proportional to the kinematic viscosity.

A decrease in viscosity of water, increase the velocity of a compound through porous medium.

viscosity of water is inversely proportional to the fall velocity.

6) Turbulency of water

Turbulency of water effect the fall velocity of water in a reservoir because the non-linearity and the zig zag path will effect the flow of water and cause the variation

in the flow. Hence more turbulency will cause decrease in fall velocity.

Question # 03:

Using any hydraulic model, explain the concept of Dimensional analysis and Similitude.

Dimensional Analysis:

It is the mathematical technique making the use of study of dimensions.

The basic principle is dimensional homogeneity which means the dimensions of each term in an equation on the both sides must be equal.

Advantages of dimensional analysis:

- ↳ less number of experiments are necessary, as opposed to the dimensional system.
- ↳ Experiments become inexpensive
- ↳ Data reduction becomes easier, single plot is sufficient to show the results.

Similitude:

It is the basic idea behind the model testing.

Dimensional Studies (analysis) and Similitude of model:

For any model, let suppose a turbine certain fluid mechanical phenomenon is governed by

$$f(\pi_1, \pi_2, \dots, \pi_n) = 0$$

where π_i are non dimensional

when the model is similar to the prototype

$$\Rightarrow (\pi_i)_{\text{model}} = (\pi_i)_{\text{prototype}}$$

where $i = 1, 2, 3, \dots, n$

↳ complete similarities require i.e

↳ Geometry similarity

↳ Kinematic similarity

↳ Dynamic similarity

Geometric Similarity

A model and prototype are geometrically similar if ~~an~~ all body dimensions in all three coordinates have the same linear scale ratio:

Kinematic Similarity

A model and prototype are kinematically similar if homologous particles is at homologous points at homologous time.

Kinematic Similarity requires geometric Similarity

Dynamic Similarities

A model and prototype are dynamically similar if ratio of any two forces are same for model and prototype

↳ we are now interested to conduct a model analysis in a wind tunnel to know the drag on the prototype.

$$n = 5, \quad k = 3, \quad m = 2$$

Repeating $\rightarrow L, \mu, \rho$

$$\pi_1 = F(L)^a (\mu)^b (\rho)^c$$

$$\pi_2 = \mu(L)^a (\mu)^b (\rho)^c$$

we will find values for a, b & c and then we see that it similar on both sides.