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:: Q1 :-

part (a)

Homogeneous :-

It states that a differential equation (of any order) is homogeneous if once all the terms involving the unknown function are collected together on one side & the equation, the other side is identically zero.

example :-

$y'' - 2y' + y = 0$ is homogeneous

Nonhomogeneous :-

nonhomogeneous second order linear equations, with standard form

$$y'' + p(t)y' + q(t)y = g(t), \quad g(t) \neq 0$$

we will focus our attention to the simpler topic & nonhomogeneous second order linear equation with constant co efficients

$$ay'' + by' + cy = g(t)$$

-: Q1 :-

Part (b)

(i) $16y'' + 24y' + 9y = 0$

$16y'' + 24y' + 9y = 0$

has characteristics equation

$16\lambda^2 + 24\lambda + 9 = 0$

$16\lambda^2 + 12\lambda + 12\lambda + 9 = 0$

$4\lambda(4\lambda+3) + 3(4\lambda+3) = 0$

$(4\lambda+3)(4\lambda+3) = 0$

$\lambda = \frac{-3}{4}, -3/4$

which are real and equal. The

The fundamental set of solution is

$x_1(t) = e^{-3/4 t}$, $x_2(t) = t.e^{-3/4 t}$. The

general solution is

$$\boxed{x(t) = c_1 e^{-3/4 t} + c_2 t e^{-3/4 t}}$$

(ii) $y'' - 4y' - 12y = 3e^{5x}$

first we find homogeneous solution

$x_h(t)$.

$\lambda^2 - 4\lambda - 12 = 0$

$\lambda^2 - 6\lambda + 2\lambda - 12 = 0$

$\lambda(\lambda-6) + 2(\lambda-6) = 0$

$(\lambda-6)(\lambda+2) = 0$

so $\lambda = -2, 6$

which are real and distinct and fundamental set of solution is

$x_1(t) = e^{-2t}$, $x_2(t) = e^{6t}$

The general solution of homogeneous is

$y_h(x) = c_1 e^{-2x} + c_2 e^{6x}$ and

Particular solution is

$y_p(x) = Ae^{5x}$

$y_p'(x) = 5Ae^{5x}$

$y_p''(x) = 25Ae^{5x}$

Substituting in given equation

$$25Ae^{5x} - 4(5Ae^{5x}) - 12Ae^{5x} = 3e^{5x}$$

$$25Ae^{5x} - 20Ae^{5x} - 12Ae^{5x} = 3e^{5x}$$

$$(25 - 20 - 12)Ae^{5x} = 3e^{5x}$$

$$-7A\cancel{e^{5x}} = 3\cancel{e^{5x}}$$

$$\boxed{A = -3/7}$$

So

$$y_p(x) = -\frac{3}{7}e^{5x}$$

Now

$$y(x) = y_h(x) + y_p(x)$$

$$\boxed{y(x) = C_1e^{-2x} + C_2e^{6x} - \frac{3}{7}e^{5x}}$$

-:Q2:-

(i) $2y'' + 5y' + 3y = 0 \qquad y(0) = 3 \quad y'(0) = 4$

$2\lambda^2 + 5\lambda + 3 = 0$

$2\lambda^2 + 3\lambda + 2\lambda + 3 = 0$

$\lambda(2\lambda + 3) + 1(2\lambda + 3) = 0$

$(2\lambda + 3)(\lambda + 1)$

$\lambda = -1, -3/2$

general solution is

$y(x) = c_1 e^{-x} + c_2 e^{-3/2 x} \rightarrow$ (A)

Put initial value problem

$y(0) = c_1 e^{-0} + c_2 e^{-3/2(0)}$

$3 = c_1 + c_2$

$c_1 + c_2 = 3$ — (1)

Take derivative of (A)

$y'(x) = -c_1 e^{-x} - 3/2\, c_2 e^{-3/2 x}$

$y'(0) = -c_1 e^{-(0)} - 3/2\, c_2 e^{-3/2(0)}$

$$-4 = -c_1 - \frac{3}{2}c_2$$

$$c_1 + \frac{3}{2}c_2 = 4 \longrightarrow 2$$

Subtract 1 and 2

$$\cancel{c_1} + c_2 = 3$$

$$\underline{-\cancel{c_1} \pm \frac{3}{2}c_2 = -4}$$

$$-\frac{1}{2}c_2 = -1$$

$$\frac{1}{2}c_2 = 1$$

$$c_2 = 2$$

Put in 1

$$c_1 + 2 = 3$$

$$c_1 = 3 - 2$$

$$c_1 = 1$$

Putting in A

$$\boxed{y(x) = e^{-x} + 2e^{-\frac{3}{2}x}}$$

(ii) $2y'' + 5y' - 3y = 0 \quad , \quad y(0) = 3 \quad , \quad y'(0) = 4$

$2\lambda^2 + 5\lambda - 3 = 0$

$2\lambda^2 + 6\lambda - \lambda - 3 = 0$

$2\lambda(\lambda+3) - 1(\lambda+3) = 0$

$(\lambda+3)(2\lambda+1) = 0$

$\lambda = -3 \, , -1/2$

$y(x) = c_1 e^{-3x} + c_2 e^{-1/2 x} \longrightarrow A$

$y'(x) = -3c_1 e^{-3x} - 1/2 \, c_2 e^{-1/2 x}$

$y(0) = c_1 e^{-3(0)} + c_2 e^{-1/2(0)}$

$3 = c_1 + c_2$

$c_1 + c_2 = 3 \longrightarrow 1$

$y'(0) = -3c_1 e^{-3(0)} - 1/2 \, c_2 e^{-1/2(0)}$

$4 = -3c_1 - 1/2 \, c_2$

$3c_1 + 1/2 \, c_2 = 4 \longrightarrow 2$

From equation 1 by 3

$3c_1 + 3c_2 = 9 \rightarrow (3)$

Subtract (2) and (3)

$3c_1 + \frac{1}{2}c_2 = 4$

$3c_1 + 3c_2 = 9$

$-\quad\quad - \quad\quad\quad -$

$-\frac{5}{2}c_2 = -5$

$c_2 = 5 \times \frac{2}{5}$

$\boxed{c_2 = 2}$

put in (1)

$c_1 + 2 = 3$

$c_1 = 1$

put in (A)

$$\boxed{y(x) = e^{-3x} + 2e^{-\frac{1}{2}x}}$$

$9\lambda^2$

(iii) $y'' - 4y' + 9y = 0 \quad y(0) = 0 \quad y'(0) = -8$

$\lambda^2 - 4\lambda + 9 = 0$

Step1 let take $m = \frac{d}{dx}$

$m^2 - 4m + 9 = 0$

if the equation is $ax^2 + bx + c = 0$

So the root for $m^2 - 4m + 9 = 0$ is

$$m = \frac{4 \pm \sqrt{16 - 36}}{2}$$

$$= \frac{4 \pm \sqrt{-20}}{2}$$

$$= 2 \pm i\sqrt{5}$$

Step2:- Hence the solution is

$$y = e^{2t}(A\cos\sqrt{5}t + B\sin\sqrt{5}t)$$

we know that

$y(0) = 0$

$0 = e^0(A)$

Hence $y = e^{2t}(B\sin\sqrt{5}t)$

Given $y'(0) = -8$

$y' = 2\sqrt{5}e^{2t} B \cos\sqrt{5t}$

$y'(0) = 2\sqrt{5}B$

$-8 = 2\sqrt{5}B$

$B = \frac{-4}{\sqrt{5}}$

The solution

$$\boxed{y = \frac{-4}{\sqrt{5}} e^{2t} \cos\sqrt{54}}$$

Q3:-

Part (A)

(1) $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$

$\mathcal{L} f(t) = 6\,\mathcal{L}(e^{-5t}) + \mathcal{L}(e^{3t})$

$+ 5\,\mathcal{L}(t)^3 - 9\,\mathcal{L}(1)$

$\Rightarrow \mathcal{L} f(t) = 6 \frac{1}{s+5} + \frac{1}{s-3} + 5 \frac{3!}{s^4}$

$-9\left(\frac{1}{s}\right)$

$$\boxed{\mathcal{L} f(t) = \frac{6}{s+5} + \frac{1}{s-3} - \frac{30}{s^4} - \frac{9}{s}}$$

(2) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

$\mathcal{L} g(t) = 4\,\mathcal{L}\cos(4t) - 9\,\mathcal{L}\sin(4t) + 2\,\mathcal{L}\cos(10t)$

$'' = 4 \frac{s}{s^2+16} - 9 \frac{4}{s^2+16} + 2 \frac{s}{s^2+100}$

$$\boxed{'' = \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}}$$

(3) $h(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

$\mathcal{L}\, h(t) = \mathcal{L}(e^{3t}) + \mathcal{L}\cos(6t) - \mathcal{L}(e^{3t})\, \mathcal{L}\cos(6t)$

$// = \frac{1}{s-3} + \frac{s}{s^2+36} - \left(\frac{1}{s-3}\right)\left(\frac{s}{s^2+36}\right)$

$\boxed{// = \frac{1}{s-3} - \frac{s}{s^2+36} - \frac{s}{(s-3)(s^2+36)}}$

-: Q3 :-

Laplace transform :-

Laplace transform is the integral transform of the given derivative function with real variable t to convert into complex function with variables. For $t \geq 0$, let $f(t)$ be given and assume the function satisfies certain conditions to be stated later on. The Laplace transform of $f(t)$ that it is denoted by $f(t)$ or $F(s)$ is defined by the equation

$$F(s) = \int_0^{+\infty} f(t) \cdot e^{-st} \cdot dt$$

Q4

(1) $y'' - 4y' = e^{3t}$ $y(0) = 0,\ y'(0) = 0$

$$\mathcal{L}(y'') - 4\mathcal{L}(y') = \mathcal{L}(e^{3t})$$

$$s^2 y(s) - s\,y(0) - y'(0) - 4s\,y(s) - y(0) = \frac{1}{s-3}$$

$$s^2 y(s) - 0 - 0 - 4s\,y(s) - 0 = \frac{1}{s-3}$$

$$s^2 y(s) - 4s\,y(s) = \frac{1}{s-3}$$

$$(s^2 - 4s)\,y(s) = \frac{1}{s-3}$$

$$y(s) = \frac{1}{(s-3)(s^2-4s)}$$

$$y(s) = \frac{1}{(s-3)(s(s-4))}$$

$$y(s) = \frac{1}{s(s-3)(s-4)}$$

Taking inverse Transform

$$y(t) = 1 - e^{3t} - e^{4t}$$

(2) $y'' + 3y' + 2y = e^{-t} \quad y(0)=0 \quad y'(0)=0$

$$\mathcal{L}(y'') + 3\,\mathcal{L}(y') + 2\,\mathcal{L}(y) = \mathcal{L}(e^{-t})$$

$$s^2 y(s) - s\,y(0) - y'(0) + 3s\,y(s) - y(0) + 2\,y(s) = \frac{1}{s+1}$$

$$s^2 y(s) - 0 - 0 + 3s\,y(s) - 0 + 2\,y(s) = \frac{1}{s+1}$$

$$s^2 y(s) + 3s\,y(s) + 2y(s) = \frac{1}{s+1}$$

$$(s^2 + 3s + 2)\,y(s) = \frac{1}{s+1}$$

$$y(s) = \frac{1}{(s+1)(s^2+3s+2)}$$

$$y(s) = \frac{1}{(s+1)(s^2+2s+s+2)}$$

$$y(s) = \frac{1}{(s+1)(s(s+2)+1(s+2))}$$

$$y(s) = \frac{1}{(s+1)(s+1)(s+2)}$$

Taking inverse Transform

$$\boxed{y(t) = e^{-t} - e^{-t} - e^{-2t}}$$