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Q1 = consider the follwing vector R^3

consider the follwing vectors in $R3$. $v1 = (1, -1, 0)$ $V_2 = (3, 2, -1)$ $V_3 = (3, 5, -2)$ (a) verify that the General vector $u = (x, y, z)$ can be written as a linear combination of V_1, V_2 and V_3 (Hint: the coefficients will be expressed as functions of the entries x, y and z of u Note: This shows that span $\{V_1, V_2 \; V_3\} = R3$ (b) can R_3 be spanned by Two vector W_1 and W_2 ?? be sure to justify.

Sol Let

Q2(b): • $T(U+V) = T(U) + T(V)$

• $T(cU) = cT(U)$

Example: Determine whether

$T: R^3 \longrightarrow R^3$ defined by

$T([x, y, z]) = [x+y, x-y, z]$

is A Linear transformation

1. Let $U [x_1, y_1, z_1]$ and

$V = [x^2\ y^2\ z^2]$ then we want

to prove $T(U+V) = T$

and

$$cT(u) = cT[x_1, y_1, z_1]$$

$$= c[x_1 + y_1, x_1 - y_1, z_1]$$

$$= [c(x_1 + y_1)\ c(x_1 - y_1)\ cz_1]$$

$$= [cx_1 + cy_1, cx_1 - cy_1, cz_1]$$

so, $T(cu) = cT(u)$

$T(U+V) = T([x_1, y_1, z_1] + [x_2, y_2, z_2])$

$= T([x_1 + x_2, y_1 + y_2, z_1 + z_2])$

$= [x_1 + x_2 + y_1 + y_2, x_1 + x_2, (y_1 + y_2)]$

Therfore $T(U+V) = T(U) +$

$T(V)$.

2. we want to prove $T(cU)$

$= cT(U)$

$T(cU) = T(c[x_1, y_1, z_1]$

$= T([cx_1, cy_1, cz_1]$

$= [cx_1 + cy_1, cx_1, cy_1, cz_1]$

Q3: Sol Determine whether or not the following sets from vector spaces over the given fields.

(a) The set V of all matrices of the form $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$ where $a, b \in R$ over R with standard addition and scalar multiplication.

Note that V is not closed under addition for $a, b, c, d \in R$ we have $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix}$ but

$$\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} + \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 2 & a+c \\ b+d & 2 \end{pmatrix} \notin V$$

we conclude that V is not a vector space with the given opreation

(b) The set V of all matrics of the form $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$ where $a, b \in R$ over R with addition and scalar multiplication defiend by

$$\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+c \\ b+d & 1 \end{pmatrix} k \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & ka \\ Kb & 1 \end{pmatrix}$$ we claim that

V is indeed a vector space with the given oprations. Note first that V is closed under the addition and scalar multiplication operation for

$\begin{pmatrix}1 & a\\ b & 1\end{pmatrix} \begin{pmatrix}1 & c\\ d & 1\end{pmatrix} \in V$ and $k \in R$

we have

$$\begin{pmatrix}1 & a\\ b & 1\end{pmatrix} \oplus \begin{pmatrix}1 & c\\ d & 1\end{pmatrix} = \begin{pmatrix}1 & a+c\\ & \end{pmatrix} \in$$

$$k \odot \begin{pmatrix}1 & a\\ b & 1\end{pmatrix} = \begin{pmatrix}1 & ka\\ kb & 1\end{pmatrix} \in V.$$

Q3 b = sol ∴ polynomials of degree n does not form a vector space becuse they don't from a set closed under.

for instance:

$$x^n - x^n = 0$$

which is not of degree n

so don't get confused with the set of polynomials of degree or equal then n+1

n+1 we often work with this space

polynomials of degree n is a set which is not closed under addition.

for example:-

if $n = 3$ then $x^3 + x^3$ and $-x^3$ are both 3rd degree polynomials but their sum is not

$$x^3 + x^2 - x^3 = x^2$$

which is not a 3rd degree polynomial.

Q4: Let M $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

inverse $ad - bc$

$$m^{-1} = \frac{1}{\det m} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = da - (bc) -$$

enteries and d for matrix M enang sd in term of put negave in front of

(b) $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$ $9 - 8 = 1$

(c) $\begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}$ $6 - 6 = 0$

(d) $A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 1 & 3 \end{vmatrix} = 3$

$1 \cdot 3 \cdot 5 + 1 \cdot 3 \cdot 4 + 1 \cdot 2 \cdot 1 - 1 \cdot 3 \cdot 4$

$1 \cdot 3 \cdot$

$15 + 8 + 7 - 12 - 10 - 7$

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