Name

Muhammad Asif

ID

16980 (16950)

Subject

Calculas and analyticalgeometry.

Teacher

Sir. Abrar

ID # 16950 Muhammad Asif

Q1a). $\dfrac{2x^3-3x^2+5}{x^2+1}$

$$\frac{\left[\frac{d}{dx}(2x^3-3x^2+5)\right](x^2+1)-2x^3-3x^2+5\left[\frac{d}{dx}(x^2+1)\right.}{(x^2+1)^2}$$

$$=\frac{(6x^2-6x)(x^2+1)-2x^3-3x^2+5(2x)+c}{(x^2+1)^2}$$

$$=\frac{6x^4-6x^2-6x^3-6x-(4x^4-6x^3+10x)+c}{(x^2+1)^2}$$

$$=\frac{6x^4-6x^2-6x^3-6x-4x^4+6x^3-10x+c}{(x^2+1)^2}$$

$$=\frac{2x^4+6x^2-16x}{(x^2+1)^2}+c$$

Scanned with CamScanner

ID # 16950 Muhammad Asif

(Q1(b)) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ with respect x

$$\frac{d}{dx}\frac{(x^2+1)^2}{x^2-1}$$

$$\Rightarrow \frac{x^2-1\frac{d}{dx}(x^2+1)^2-(x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$\Rightarrow \frac{x^2-1\frac{d}{dx}(x^4+2x^2+1)-(x^2+1)^2(2x-0)}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1)(4x^3+4x+0)-(x^2+1)^2 2x}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1)(4x^3+4x)-(x^2+1)^2 2x}{(x^2-1)^2}$$

$$= \frac{4x^5-\cancel{4x^3}+\cancel{4x^3}-4x-2x^3+2x-4x^3}{(x^2-1)^2}$$

$$= \frac{4x^5-6x^3-2x}{(x^2-1)^2}$$

dx

ID # 16950 Muhammad Asif

Q2a) Find $\frac{dy}{dx}$ if $y = (+2\sqrt{x})^3 x^{2/3}$

As chain Rule is

$[f(x)^n]' = n f(x)^{n-1} f(x)'$

$y \ (2\sqrt{x}+1)^3 x^{2/3}$

$\frac{d}{dx} y = \frac{d}{dx}\left[(2\sqrt{x}+1)^3 x^{2/3}\right]$

$= x^{\frac{2}{3}} \frac{d}{dx}\left[(2\sqrt{x}+1)^3\right] + (2\sqrt{x}+1)^3 \frac{d}{dx}\left[x^{2/3}\right]$

Now apply chain Rule

$= x^{2/3} \, 3(2\sqrt{x}+1)^{3-1} \frac{d}{dx}(2\sqrt{x}+1) + (2\sqrt{x}+1)^3 \frac{2}{3} x^{\frac{2}{3}-1} \frac{d}{dx} x$

$= x^{\frac{2}{3}} \, 3(2\sqrt{x}+1)^3 \, 2x^{-1/2} + (2\sqrt{x}+1)^3 \frac{2}{3} x^{-1/3}$

$= 3x^{\frac{2}{3}-\frac{1}{2}} (2\sqrt{x}+1)^2 + \frac{2(2\sqrt{x}+1)^3}{3 \sqrt[3]{3}}$

$= 3\sqrt[6]{x} \, (2\sqrt{x}+1)^2 + \frac{2(2\sqrt{x}+1)^3}{3\sqrt[3]{x}}$

$= \frac{3\sqrt[3]{x}\left[3\sqrt[6]{x}(2\sqrt{x}+1)^2\right] + 2(2\sqrt{x}+1)^3}{3\sqrt[3]{x}}$

$\frac{d}{dx} y = \frac{(2\sqrt{x}+1)^2 (13\sqrt{x}+2)}{3\sqrt[3]{x}}$

Scanned with CamScanner

ID# 16950 Muhammad Asif

Q 2b) Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1-x}{1+x}}$

As chain rule is

$[f(x)^n]' = n\, f(x)^{n-1} f'(x)$

Hence

$$y = \left(\frac{1-x}{1+x}\right)^{1/2}$$

$$\frac{d}{dx} y = \frac{d}{dx}\left[\left(\frac{1-x}{1+x}\right)^{1/2}\right]$$

using chain rule

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{1/2 - 1} \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\left[\frac{\frac{d}{dx}(1-x)\,(1+x) - (1-x)\frac{d}{dx}(1+x)}{(x+1)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2}\left[\frac{-1(1+x) - (1-x)(1)}{(x+1)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2}\left[\frac{-1-x-1+x}{(x+1)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{1/2}\left[\frac{-2}{(x+1)^2}\right]$$

$$= -1\,\frac{(1+x)^{1/2}}{(1-x)^{1/2}}\,\frac{1}{(1+x)^2}$$

$$= \frac{-1}{(1-x)^{1/2}(1+x)^{2-1/2}}$$

$$\frac{dy}{dx} = \frac{-1}{(1-x)^{1/2}(1+x)^{3/2}}$$

ID # 10950 Muhammad Asif

Q.3(a) $\int \frac{1}{\sqrt[2]{x^3}} dx$

$= \int \frac{1}{x^{3/2}} dx$

$= \int x^{-3/2} dx$ $\quad a^{\frac{1}{m}} = a^{-m}$

$= \int x^{-3/2} dx$

$= \frac{x^{-3/2+c}}{\frac{-3}{2}+1}$ $\quad \int x^m dx = \frac{x^{m+1}}{m+1}$

$= \frac{x^{\frac{-c}{2}}}{-1/2}$

$= -2 \cdot \frac{1}{x^{1/2}}$

$= \frac{-2}{\sqrt{x}} + c$

ID # 16950 Muhammad Asif

Q3(b) find the integration of

$$\int \frac{1}{(6x+7)^6}\,dx$$

$$= \int \frac{1}{(6x+7)}\,dx \quad \text{—(i)}$$

let $u = 6x+7$ —(2)

Apply derivative wrt x

$$\frac{du}{dx} = 6 \rightarrow (3)$$

$$dx = \frac{1}{6}\,du \rightarrow (4)$$

put (2)=(4) in equation (1)

$$= \frac{1}{6}\int \frac{1}{u}\,du$$

$$= \frac{1}{6}\ln|u| + c$$

$$= \frac{\ln|u|}{6} + c$$

= putting value of u.

$$= \frac{\ln(6x+7)}{6} + c.$$

Scanned with CamScanner