Department of Electrical Engineering
Final Exam
Assignment Date: 27/06/2020 **Course Details**

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	50

Student Details

Name: Syed M Zahoor **Student ID:** 12595

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks** **7** **CLO** **2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks** **7** **CLO** **2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks** **6** **CLO** **2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks** **6** **CLO** **2**
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\lvert H(\frac{\pi}{4})\rvert^2 = \frac{1}{2}$.	**Marks** **6** **CLO** **3**

<table>
<tr><td></td><td>(b)</td><td>Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.</td><td>Marks
6
CLO
3</td></tr>
<tr><td rowspan="2">Q 4</td><td>(a)</td><td>A finite duration sequence of Length L is given as
$$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$$
Determine the N- point DFT of this sequence for N ≥ L</td><td>Marks
6
CLO
2</td></tr>
<tr><td>(b)</td><td>Perform the circular convolution of the following two sequences. Solve the problem step by step
$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$
$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$</td><td>Marks
6
CLO
2</td></tr>
</table>

Name : Syed. M. Zahoor DSP

ID 12595 27/06/2020

Q(a) 1 Determine the response $y(n), n \geq 0$, of the system described by the second order difference equation

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$ and the initial conditions are $y(-1) = y(-2) = 0$

Solution

The characteristic equation

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \text{ Hence}$$

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

There particular solution

$$y_p(n) = k(-1)^n u(n).$$

Substituting the solution into the difference equation we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$$

$$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

(2)

for $n=2$, $k(1+4+4)=2 \Rightarrow k=\frac{2}{9}$

The total Solution is

$$y(n)=\left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right]u(n)$$

from The Initial conditions, we obtain $y(0)=1$, $y(1)=2$ then

$$c_1+\frac{1}{9}=1$$

$$\Rightarrow c_1=\frac{7}{9}$$

$$2c_1+2c_2=\frac{2}{9}=2$$

$$\Rightarrow c_2=\frac{1}{3}$$

Q(1)
(B)

Solution

the characteristic equation

$$\lambda^2-0.7\lambda+0.1=0$$

$$\lambda=\frac{1}{2},\frac{1}{5}$$

(3)

$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$

With $x(n) = \delta(n)$ we have

$$y(0) = x(0) = 2$$

$$y(1) = 0.7y(0) = \Rightarrow (1) = 1.4,$$

Hence $c_1 + c_2 = 2$ {

$$\frac{1}{2} c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5} c_2 = \frac{14}{5}$$

The equation yield

$$c_1 = \frac{10}{3}, \quad c_2 = \frac{-4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$$

The step response as

$$Q(n) = \sum_{n=0}^{n} h = (n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}^n\left(2^{n+1} - 1\right)u(n) - \frac{1}{3}\left(\frac{1}{5}\left(5^{n+1} - 1\right)u(n)\right.\right.$$

(a) (4)

Q2 Determine the Causal Signal $x(n)$ having the Z-transform

$$X(Z) = \frac{1}{1-2Z^{-1}(1-Z^{-1})^2}$$

Take Invere Z-Transform using partial fraction method:

Solution

$$X(Z) = \frac{1}{(1-2Z^{-1})(1-Z^{-1})^2}$$

As we know that

$$= \frac{A}{(1-2Z^{-1})} + \frac{B}{(1-Z^{-1})} + \frac{C}{(1-Z^{-1})^2}$$

$$A = 4, \quad B = -3, \quad C = -1$$

Hence, $x(n) = [4(2)^n - 3 - n]\,u(n)$

Q2 (b) Evaluate The Inverse Z-transform using The Complex inversion Integral.

$$X(Z) = \frac{1}{1-aZ^{-1}} \qquad |Z| > |a|$$

Solution:

We have

$$X(n) = \frac{1}{2\pi j}\oint_c \frac{Z^{n-1}}{1-aZ^{-1}}\, dZ$$

$$= \frac{1}{2\pi j}\oint_{c} \frac{Z^n\, dZ}{Z-a}$$

where c is a circuit at radius greater than |a|. we shall evaluate this Integral using with $f(Z) = Z^n$

We distinguish two Cases.

① If $n \geq 0$ $f(x)$ has only Zeros and hence no poles inside c. The only inside c is $Z = a$ hence.

② If $n < 0$. $f(Z) = Z^n$ has an nth order pole at $Z = 0$ which also Inside C.

Thus there are contributions from both poles. for $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz$$

$$= \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

if $n = -2$ we have

$$x(-2) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)} dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$

Thus

$$x(n) = a^n u(n)$$

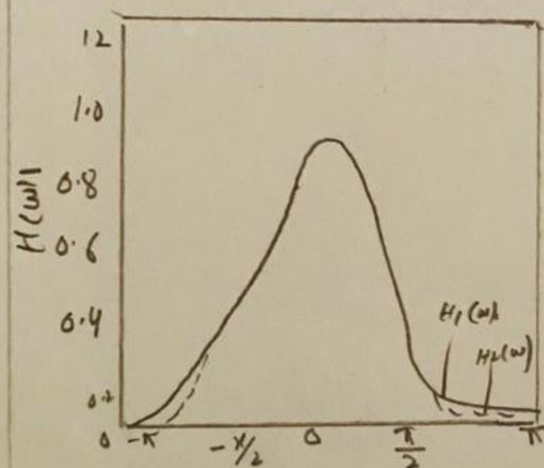
Q3 (a) A two pole low pass filter has the System response

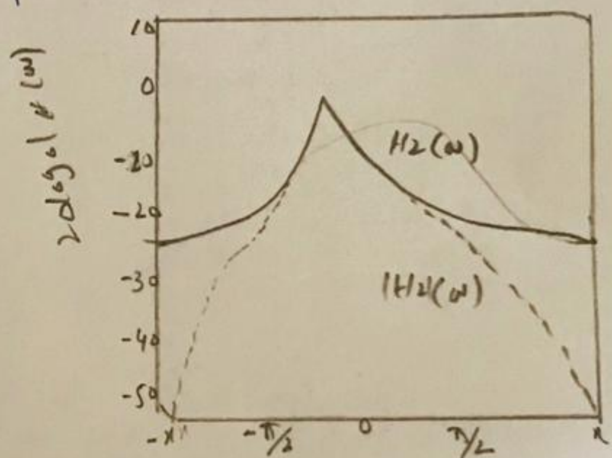
$$H(z) = \frac{b_0}{(1 - pz^{-1})^2}$$

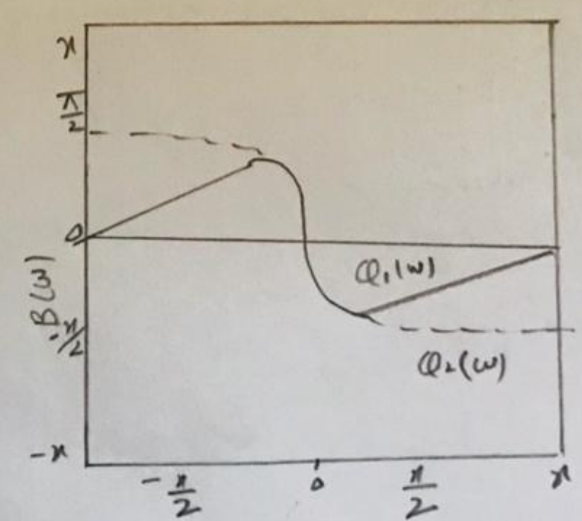
$$H(0) = 1 \text{ and } |H(\tfrac{\pi}{4})|^2 = \tfrac{1}{2}$$

Solution:

A two lowpass filter







Now The values b_0 and p

$$H(0) = 1$$

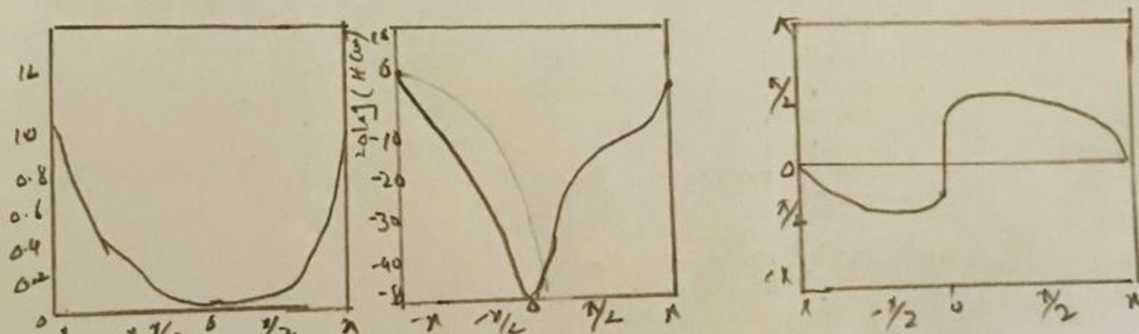
$$|H(\pi/4)|^2 = \frac{1}{2}$$

At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$



At $= \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{1 - pe^{j\pi(4)/2}}$$

$$= \frac{1-p^2}{1 - p\cos(\pi/4) + jp\sin(\pi/4)^2}$$

$$= \frac{(1-p)^2}{(1 - p\sqrt{2} + jp/\sqrt{2})^2}$$

Hence

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]^2}=\frac{1}{2}$$

or Equivalently

$$\sqrt{2}(1-p)^2=1+p^2-\sqrt{2}p$$

The value of $p=0.32$ Satisfies the Equation. Consequenty. The System function for The desired filter is

$$H(z)=\frac{0.46}{(1-0.32z^{-1})^2}$$

Q3 B

Solution :

Give Data :

$\omega = \pi/2$

$\omega = 0$ and $\omega = \pi$

$1/\sqrt{2}$ at $\omega = 4\pi/9$

By The filter requirement:

$$\text{Poles} = P_{1,2} = re^{\pm j\pi/2}$$

$$\text{Zeroes} = Z_{1,2} = \pm 1$$

$$\Rightarrow H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

$$= G = \frac{2}{-1+r^2}$$

$$= G = \frac{1-r^2}{2}$$

To set r use $H\frac{4\pi}{9} = 1/\sqrt{2}$

requirement.

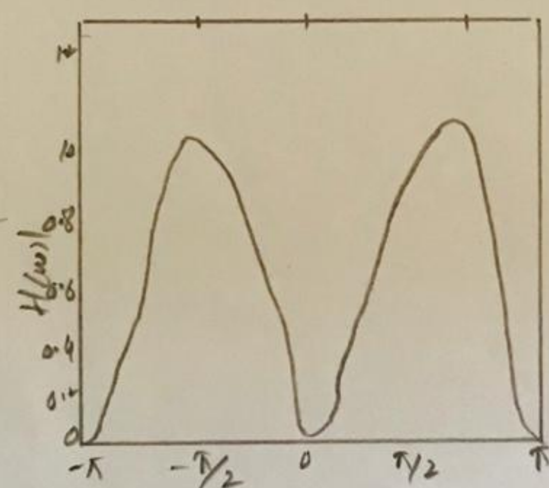
Now

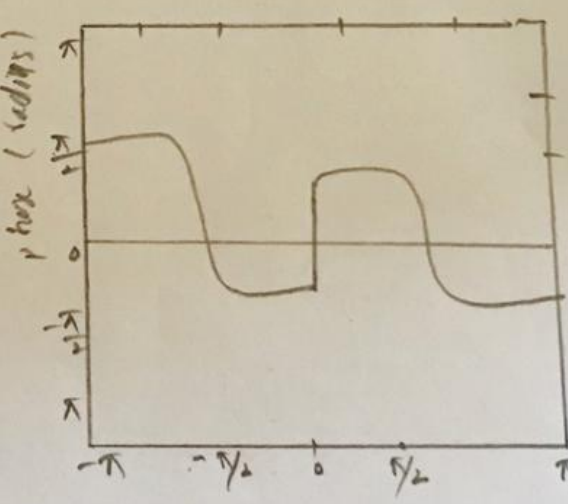
$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

Evaluating gives $r^2 = 0.7$ Therefore

$$H(Z) = 0.15\,\frac{1-Z^{-2}}{1+0.7Z^{-2}}$$





Q(4)

(A) A finite duration Sequence of Length L given as

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determine The point DFT of This Sequence for $N \geq L$

Solution:

As we know that

The fourier Transform of This Sequence is

$$X(\omega) = \sum_{n=1}^{L-1} x(n) e^{-j\omega}$$

$$= \sum_{n=1}^{L=1} e^{-j\omega} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j2\pi K L/N}$$

for $L=10$ The N point DFT of $x(n)$ is simple $X(\omega)$ evaluated at the set of N equally spaced frequencies $\omega_k = 2\pi K/N$, $K = 0, 1 \ldots N-1$ Hence

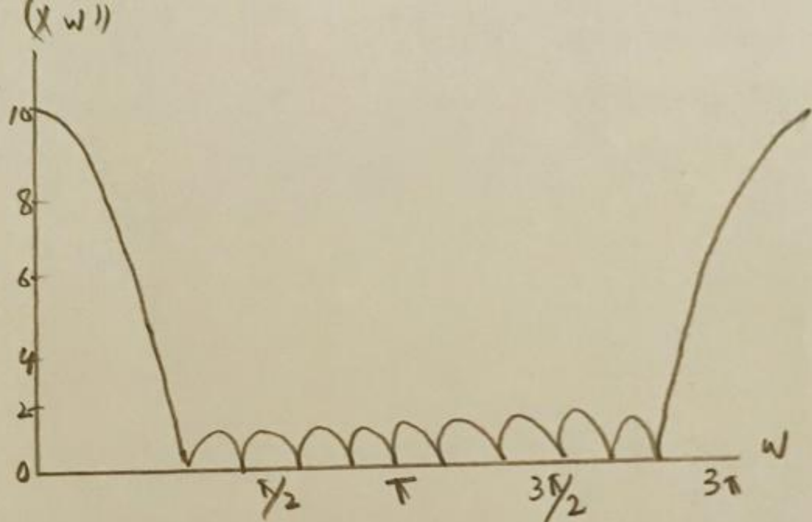
(13)

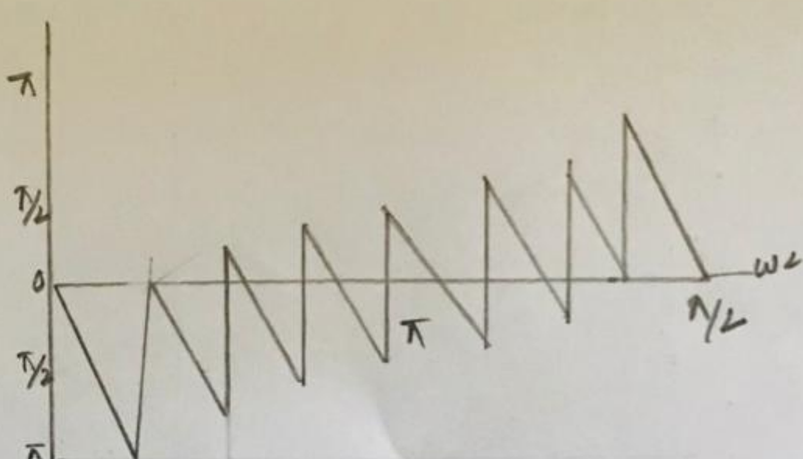
Hence

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{j2\pi K/N}}$$

$k = 0, 1 \ldots N-1$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





2(a) (B) Perform The circular convolution of The following two Sequence. Solve The problem step by Step:

$$x_1(n) = \{\underset{\uparrow}{2}\ 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}\ 2, 3, 4\}$$

Solution:

Each Sequence consists of four non zero point.

now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ $x_2(n)$ as specified by Begining with $m=0$ we have

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N$$

$x_2((-n))_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle as illustrated in other words The folded sequence is simple $x_2(n)$ graphed in a clockwise direction

$$x_3(0) = 4$$

for $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4$$

$x_2((1-4))_4$

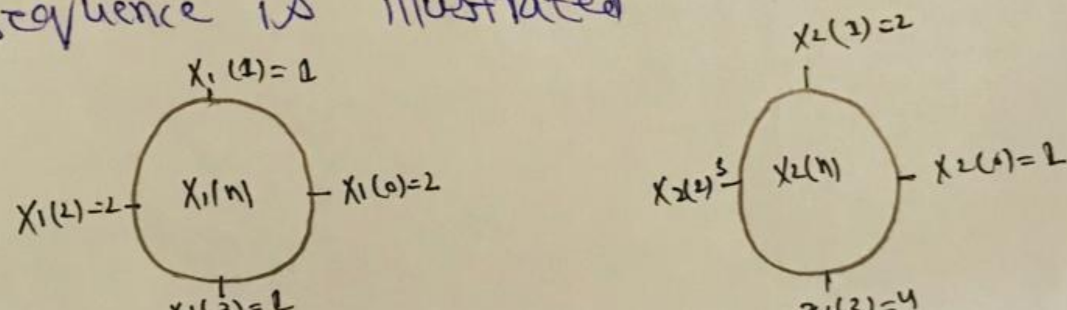
$x_2((-n))_4$

$x_3(1) = 16$

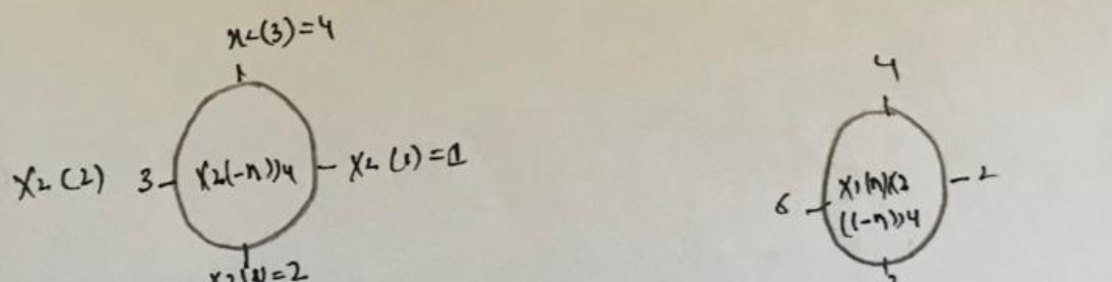
for $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

now $x_2((2-n))_4$ is folded sequency rotated two unit of time in the counterclockwise direction. The resultant sequence is illustrated



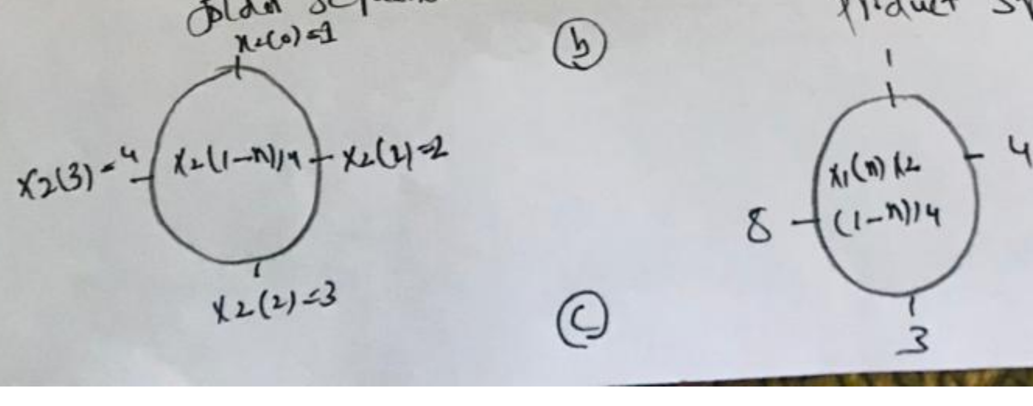
(a)



folded sequence

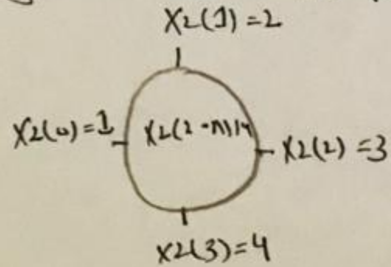
product sequence

(b)



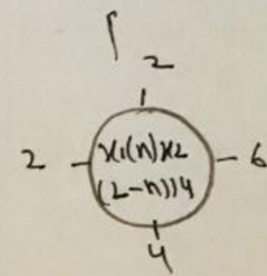
(c)

folded sequence rotated by one unit in time.

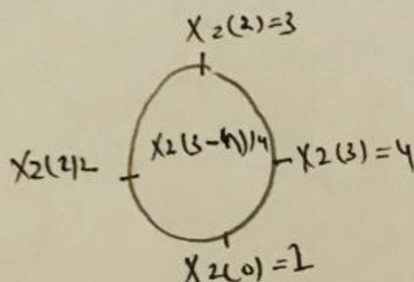


(c)

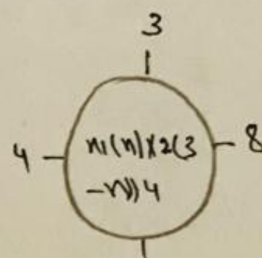
Product Sequence



product sequence



(d)



folded sequence rotated by Three unit in time.

(e)

product squence

along with The product Sequence $x_1(n)\,x_2(2-n))_4$

By Summing The foure terms in the product Sequence, we obtain

$$x_3(2)=14$$

for $m=3$ we have

$$x_3(3)=\sum_{n=0}^{3} x_1(n)\,x_2(3-n))_4$$

$x_2((-n))_4$

$x_2(3-n))_4$

Now

$$x(3)=16$$

$m=3$

$$x_3(n)=\{14,16,14,16\}$$