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Q #01

Sol:

$$f(t) = 1+t \quad -\pi \le t \le \pi$$

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{x} a_n \cos t + \sum_{n=1}^{x} b_n \sin t \to (1).$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\,dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\,dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2}\right)\right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2\pi} (2\pi + \pi^2)}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi} (1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin ntd}{n\,dt} (1+t)\right)$$

$$a_n = \frac{1}{\pi} \left((1+t) \frac{\sin nt}{n} - \frac{\cos nt}{n^2} \Big|^{\pi} \right)$$

$$a_n = \frac{-1}{n^2\pi} \left(\cos n\pi - \cos n(-n)(-\pi)^{-\pi} \right.$$

$$a_n = \frac{-1}{n^2\pi}(-1-(-1))$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\, dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt\ d/dt(1+t)dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.\frac{(1+t)(-\cos nt)}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.-\frac{(1+t)(\cos nt)}{n}\right|_{-\pi}^{\pi} + \left(\left.\frac{\overset{0}{\sin n\pi}}{n^2}\right|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - ((1+(\pi))(\cos n(\pi)\right)$$

$$b_n = \frac{-1}{n\pi}(\cancel{\cos n\pi} + \pi\cos\pi - \cancel{\cos n\pi} + \pi\cos n\pi)$$

$$b_n = \frac{-1}{n\cancel{\pi}}(2\cancel{\pi}\cos n\pi)$$

$$\text{Here } \cos n\pi = \frac{(-1)^{n+1}}{n}$$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

so equation become

$$\boxed{f(x) = \frac{1}{2\pi}(2\pi+\pi^2) + 0 + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin t}$$

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Q # 02

Ans:

$$A = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix}$$

Eigen values = ?

Sol:

step ①

we have

$(A - \lambda I)X = 0$ A = Given matrix

step # 02

we have the characteristic equation is given by

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

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Step # 03 :-

$$\lambda^3 - \left|\begin{array}{c}\text{sum of diagonal}\\ \text{elem}\end{array}\right| \lambda^2 + \left|\begin{array}{c}\text{sum of diagonal}\\ \text{minor}\end{array}\right| \lambda - |A| = 0 \rightarrow (B)$$

sum of Diagonal Elements $\Rightarrow 1+1+2 = 4$

sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$\Rightarrow (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B)

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \rightarrow (C)$

$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix}$

$+1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

$\Rightarrow 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

By putting values in (c)

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

using quadratic formula.

$a = 1$
$b = -4$
$c = -3$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{20}}{2}$$

$$\lambda = \frac{4 + \sqrt{20}}{2} = \frac{4 - \sqrt{20}}{2}$$

we have eigen values

$$\boxed{\lambda = \left(0, \frac{4+\sqrt{20}}{2}, \frac{4-\sqrt{20}}{2}\right)}$$

(Required solution)

Q #03

Sol:

Solve the following system of linear equation.

$5x + 0 + 4z + 2m = 3$

$x - y + 2z + m = 1$

$4x + y + 2z + 0 = 1$

$x + y + z + m = 0$

Sol:

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad R_4 - R_2$$

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1+4/5 \\ 0 & -1 & +6/5 & +4/5 \\ 0 & 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 3/5 \\ -1 \end{bmatrix} \quad -1/5 \times R_3$$

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$$\left(\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right)$$ $5 \times R_3$ and $5 \times R_4$

$$\left(\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right)$$ $5R_3$ and $5R_4$

$$\left(\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right)$$ $1/5 \times R_1$

$$\left(\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right)$$ $R_2 \times 5$

$$\left(\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right)$$ $R_3 - R_2$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_2 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad R_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 120/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$(x, y, z, m) = (3/4, 31/21, -11/21, 1/3)$

$x = 3/4$

$y = 31/21$

$z = -11/21$

$m = 1/3$

Q #04 :

verify that

$$u(x,t) = \sin(x+2t)$$

is a solution of a one dimensional equation.

Sol :- Given that

$$u(x,t) = \sin(x+2t)$$

Differential w.r.t. x partially.

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\ \partial/\partial x\,(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \cdot \partial/\partial x\,(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)}$$

and $u(x,t) = \sin(x+2t)$

D.P. w.r.t. t

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)\,(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t)\,(0+2)$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

we know that one dimensional eq. is

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

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For the orbitery constant $C = \pm 2$

$$-4 \sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4 \sin(x+2t) + 4 \sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verify

for the orbitrary constant

$$C = 2$$