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SEC : A

Subject : Differential equation

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(1)

Q No 1

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \quad y(0)=0$$

Sol:- $\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \quad y(0)=0$

As we know that

$y(0)=0$ so $X=0$ $y=0$

$dy = e^{y} e^{-t} \sec(y)(1+t^2)\, dt$

$\frac{1}{e^{y} \cdot \sec(y)} dy = (1+t^2) e^{-t} dt$

As $\cos(y) = \frac{1}{\sec(y)}$

$\int e^{-y} \cos y\, dy = \int (1+t^2) e^{-t} dt$

using integration by part

$e^{-y} \int \cos y\, dx - \int (\int \cos y \cdot \frac{d}{dy} e^{-y})\ (1+t^2)$

$\int e^{-t} - \int (\int e^{-t} \frac{d}{dt} (1+t^2)$

eq (1)

L.H.S

$e^{-y} \int \cos y\, dx - \int (\int \cos y \cdot \frac{d}{dy} e^{-y})$

(P.T.O)

(2)

$e^{-y}\sin y - \int (\sin y \cdot e^{-y}(-1))$

$e^{-y}\sin y + \int (\sin y \cdot e^{-y})$

$e^{-y}\sin y + \int (e^{-y}\sin y)$

Again using Integration by parts

$e^{-y}\sin y + e^{-y}(-\cos y) - \int (\int \sin y \, \frac{d}{dy} e^{-y})$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int (-\cos y \, \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y - \int (\cos y \, e^{-y})$

Since $\int (\cos y e^{-y}) = L.H.S$

Since it is again Same to the First one so L.H.S will become

$LHS = e^{-y}(\sin y - \cos y) - L.H.S$

$2LHS = e^{-y}(\sin y - \cos y)$

$$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$$

(3) Now taking R.H.S

$\int (1+t^2)\, e^{-t}\, dt$

$\Rightarrow (1+t^2) \int e^{-t} - \int (\int e^{-t} \frac{d}{dt} (1+t^2))$

$\Rightarrow (1+t^2)\, e^{-t} \int (-e^{-t} (2t))$

$\Rightarrow -(1+t^2)\, e^{-t} + \int (2t) e^{t})$

Again using integration by parts

$\Rightarrow -(1+t^2)\, e^{-1} + (2t \int e^{-t} - \int (\int e^{-t} \frac{d}{dt}\, 2t))$

$\Rightarrow -(1+t^2)\, e^{-t} + (-2t\, e^{-t} - \int (-e^{-t}\, 2)$

$\Rightarrow -(1+t^2) e^{-t} + (-2t\, e^{-t} + \int (2e^{-t})$

$\Rightarrow -(1+t^2) e^{-t} + (-2t\, e^{-t} - 2e^{-t}) + C$

$\Rightarrow -(1+t^2) e^{-t} - 2te^{-t} - 2e^{-t} + C$

$\Rightarrow -e^{-t} - e^{-t} t^2 - 2te^{-t} - 2e^{-t} + C$

$\Rightarrow -(t^2+2t+3)e^{-t} + C = R.H.S$

Now take L.H.S = R.H.S

$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + C$

we know that.

(4)

$t=0 \qquad y=0$

Put it above

$$\Rightarrow \frac{1}{2}(0-1) = -3 + c$$

$$c = 5/2$$

put value of c

$$\boxed{\frac{e^{-y}}{2}(\sin y - \cos y) = -(x^2+2+3)e^{-t} + \frac{5}{2}}$$

Ans

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(1) Question No # 2

$$(\sqrt{x+y}) + \sqrt{(x-y)}\,dx - (\sqrt{x+y}) - \sqrt{(x-y)}\,dy = 0$$

Sol ⇒ $(\sqrt{x+y}) + \sqrt{(x-y)}\,dx - (\sqrt{(x+y)} - \sqrt{(x-y)})\,dy = 0$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{(x+y)} - \sqrt{x-y})} \quad \text{—} \quad (1)$$

This is Homogeneous Differential eq in x and y to solve this put

$$y = vx$$

This eq (1) becomes

$$v + x \cdot \frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx})}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{(1+v)} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1 \cancel{+v} + 1 \cancel{-v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}(v)}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

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(2)

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking integral on b/s

$$\int \frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$\Rightarrow \frac{1}{2}(1-v^2)^{-\frac{1}{2}}(-2v)(dv = dt$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

P.T.O

(3) $\ln(1+\sqrt{1-v^2}) = -\ln cx$

$\ln(1+\sqrt{1-v^2}) = \ln((cx)^{-1}$

$1+\sqrt{1-v^2} = \frac{1}{cx}$

$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$

$1+\frac{\sqrt{x^2-y^2}}{x^2} = \frac{1}{cx}$

$x+\sqrt{x^2-y^2} = 1/c$

$x+\sqrt{x^2-y^2} = c_1 \quad \because 1/c = c_1$

which is Required solution

———x ———x ———x

(1) Question No # 3

$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

Solution :-

$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

$\Rightarrow f(D)y = f(x)$

As it is non-homogenous linear equation so solution will be

$y = y_c + y_p$ — (i)

$D^4 - D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow \boxed{D = 0}$

$D^2 + 1 = 0 \Rightarrow D^2 = -1$

$D = \sqrt{-1} \Rightarrow D = i$ or $D = \boxed{0+i}$

Root are real and complex

$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x$

$y_p = \frac{1}{f(D)} f(x)$

(2) $y_P = \frac{1}{D^4 + D^2}(3x^2 + 4\sin x - 2\cos x)$

$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$

$f(D) = D^4 + D^2$

at $D=0 \Rightarrow f(D) = 0$

So $f'(D) = 4D^3 + 2D$

Now also for $D=0 \Rightarrow f'(D) = 0$

again Differentiating

$f''(D) = 12D + 2$

So for $D = 0$

$f''(0) = 12(0) + 2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$\Rightarrow y_P = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2\, 2\cos x}{12D+2}$

Putting $D=0$ in all

P.T.O

(3) So

$$y_P = \frac{x^2 \; 3x^2}{12(0)+2} + \frac{x^2 \, 4\sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$y_P = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

So Putting in equation (i)

$$y = c_1 + c_2 \cos x + c_3 \cos x + \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$

Ans