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Q No 1 (part a)

Ans

Given: $\int g \sqrt[4]{1-g^2}\, dg$

<u>Solution</u>

Let

$$1-g^2 = u$$

$$\frac{d}{dg}(1-g^2) = \frac{d}{dg}u$$

$$-2g = \frac{du}{dg}$$

$$g\,dg = \frac{-1}{2}du$$

$$\text{Now} = \int (u)^{1/4} \cdot \left(-\frac{1}{2}\right) du$$

$$= \frac{-1}{2}\int u^{1/4}\, du \qquad \because \frac{1}{4}+1 = \frac{5}{4}$$

$$= \frac{-1}{\cancel{2}_1} \cdot \frac{\cancel{4}^2}{5}\, u^{5/4} + c$$

$$= \frac{-2}{5} u^{5/4} + C$$

By back substitution

$$= \frac{-2}{5}(1-g^2)^{5/4} + C$$

$$\left( = \frac{-2}{5}(1-g^2)^{5/4} + C \quad \text{Ans.} \right)$$

Q No 1 (part b)

Ans. Given: $\int_0^1 x^3(1+x^4)^3\,dx$

Solution: Let

$$1+x^4 = u \quad \text{---} (1)$$

$$\frac{d}{dx}(1+x^4) = \frac{d}{dx}u$$

$$4x^3 = \frac{du}{dx}$$

$$x^3\,dx = \frac{1}{4}\,du$$

Now put $x=0$ in eq (1)

$$1+0^4 = 1$$

$$0 = 1$$

& put $x=1$ in eq (1)

$$1+(1)^4 = 2$$

$$u = 2$$

$$= \int_1^2 (u)^3 \frac{1}{4}\,du$$

$$= \frac{1}{4}\int_1^2 u^3\,du$$

$$= \frac{1}{4}\frac{u^4}{4}\Big|_1^2$$

$$= \frac{1}{4}\left(\frac{(2)^4}{4} - \frac{(1)^4}{4}\right)$$

$$= 3/8$$

$$\boxed{= 3/8 \text{ Ans}}$$

Q 2 Part (a)

Find the center of sphere and radius !

$$x^2+y^2+z^2+3x-4z+1=0$$

Solutions

$$x^2+y^2+z^2+3x-4z+1=0$$

$$(x^2+3x)+y^2+z^2-4z+1=0$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2-4z+\left(\frac{-4}{2}\right)^2\right)$$

$$= -1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$(x+3/2)^2+(y+0)^2+(z-2)^2=\frac{21}{4} \Rightarrow So\ (x_0, y_0, z_0) \Rightarrow center$$

$$= \left(-\tfrac{3}{2}\right), 0, 2)$$

( find Radius as $\sqrt{\frac{21}{4}}$ )

Q2 Part (b)

Ans:

Given :: $y=\sqrt{x}$

$0 \le x \le 4 \Rightarrow a \le x \le b$

Solution: we know that

$$V=\int_0^4 \pi y^2 dx$$

$$V=\int_0^4 \pi (\sqrt{x})^2 dx$$

$$V=\pi \int_0^4 x\,dx$$

$$V=\pi \left.\frac{x^2}{2}\right|_0^4$$

$$V=\frac{\pi}{2}(4^2-0) \Rightarrow \frac{\pi}{\cancel{2}} \cancel{16}\,8$$

( $V=8\pi$ Ans )

Q 3 = If $A = 2i - 4j + \sqrt{5}k$ and
$B = -2i + 4j - \sqrt{5}k$
Then illustrate the vector $Proj_A B$

Solutions

We have projection formula;

$$Proj_A B = \frac{B \cdot A}{\|A\|^2} A \rightarrow (1)$$

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$B \cdot A = (-(4)(i \cdot i) - 16(j \cdot j) - (5)(k \cdot k))$$

We know That

$i \cdot i = j \cdot j = k \cdot k = 1.$

$$B \cdot A = (-4(1) - 16(1) - 5(1)).$$

$$= -4 - 16 - 5$$

$$B \cdot A = -25 \rightarrow (a)$$

$$\|A\|^2 = \left(\sqrt{(2i)^2 + (-4j)^2 + (\sqrt{5}k)^2}\right)^2$$

$$= \left(\sqrt{4 + 16 + 5}\right)^2$$

$$\|A\|^2 = 25 \rightarrow (b)$$

$$A = 2i - 4j + \sqrt{5}k \rightarrow (c)$$

put (a), (b) & (c) in Eq (1)

we got

$$Proj_A B = \frac{-25}{25}(2i - 4j + \sqrt{5}k).$$

$$// = -1(2i - 4j + \sqrt{5}k)$$

$$\Rightarrow \boxed{Proj_A B = -2i + 4j - \sqrt{5}k.}$$

Q4 =

Given $y = -x^2 + 5x - 4 \quad [0, 2]$

Required:

Area = ?

Solution:

As $a = 0$

$b = 2$

$A = \int_a^b f(x)\,dx$

$A = \int_0^2 (-x^2 + 5x - 4)\,dx$

$A = \left(-\frac{x^3}{3} + \frac{5}{2}x^2 - 4x\right)\Big|_0^2$

$A = -\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2) - (0)$

$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8\right.$

$A = \frac{-8}{3} + \frac{\cancel{20}^{10}}{\cancel{2}} - 8$

$\frac{-8}{3} + 2$

$A = -\frac{2}{3} = -0.6$

As area is never in negative so we take the value positive.

(Ans) $\boxed{A = 0.6}$

Q5 part (a): Estimate the angle between $A = i - 2j - 2k$ and $B = 6i + 3j + 2k$

Solution: To find the angle b/w any two vectors we have a formula:

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{\|A\|\,\|B\|}\right) \rightarrow (1)$$

$A \cdot B = (i - 2j - 2k) \cdot (6i + 3j + 2k)$

$A \cdot B = (6(i \cdot i) - 6(j \cdot j) - 4(k \cdot k))$

put $i \cdot i = j \cdot j = k \cdot k = 1$

$A \cdot B = 6(1) - 6(1) - 4(1)$

$= 6 - 6 - 4$

$\boxed{A \cdot B = -4} \rightarrow (a)$

$\|A\| = \sqrt{(1)^2 + (2)^2 + (2)^2} = \sqrt{1+4+4} = \sqrt{9}$

$\boxed{\|A\| = 3} \rightarrow b$

$\|B\| = \sqrt{(6)^2 + (3)^2 + (2)^2} = \sqrt{36+9+4} = \sqrt{49}$

$\boxed{\|B\| = 7} \rightarrow (c)$

put (a), (b) & (c) in eq (1)

$\theta = \cos^{-1}\left(\frac{-4}{(3)(7)}\right)$

$\boxed{\theta = \cos^{-1}\left(\frac{-4}{21}\right)}$

By calculator

$(\theta = 100.98^\circ)$

Q5 part(b)

Change into a spherical coordinate for the sphere:

$$x^2+y^2+(z-1)^2=1$$

Solution

$$x^2+y^2+(z-1)^2=1$$

$$(\rho\sin\phi\cos\theta)^2+(\rho\sin\theta\sin\theta)^2+(\rho\cos\theta-1)^2=1$$

$$\rho^2\sin^2\theta\cos^2\theta+\rho^2\sin^2\theta\sin^2\theta+\rho^2\cos^2\theta+1-2\rho\cos\theta=1$$

$$\rho^2\sin^2\theta(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\theta+1-2\rho\cos\theta=1$$

$$\rho^2(\sin^2\theta)+\rho^2\cos^2\theta-2\rho\cos\theta=1-1$$

$$\rho^2(\sin^2\theta+\cos^2\theta)-2\rho\cos\theta=0$$

$$\rho^2=2\rho\cos\theta$$

$$\rho=2\cos\theta$$