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SECTION A.

PAPER D. EQUATION.

DATE 31-06-2020.

QUESTION NO 1 :

(P1)

PART 1 :- $w = \sin(x+ct) + \cos(2x+2ct)$

SOLUTION :-

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c(-\sin(2x+2ct)) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \rightarrow (1)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+2ct)\right]$$

$$\boxed{c^2 \cdot \frac{\partial^2 w}{\partial x^2} \quad \text{Ans.}}$$

PART II:-

SOLUTION:- $w = \tan(2x+ct)$

Now $\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$

& $\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x+xt)\right)$

$= c2^2 2\sec^2(2x+ct)\tan(2x+ct$

Now $\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$

$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$

$= 4c^2\sec^2(2x+ct)\tan(2x+ct) = 4c^2\sec^2(x+ct)$

$\tan(2x+ct)$

$0=0$ Satisfied.

Q.No 2: $F(x) = x \; ; \quad -\pi < x \leq 0$

$2x \; ; \quad 0 \leq x \leq \pi$

$$F(x) = \begin{cases} x \; ; & -\pi < x \leq 0 \\ 2x \; ; & 0 \leq x \leq \pi \end{cases}$$

Now we will find a_0, a_n & b_n

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi} \int_{-\pi}^{0} x\,dx + \frac{1}{\pi} \int_{0}^{\pi} 2x\,dx.$$

$$a_0 = \frac{1}{\pi} \left[\frac{x^2}{2} \right]_{-\pi}^{0} + \frac{2}{\pi} \left[\frac{x^2}{2} \right]_{0}^{\pi}$$

$$a_0 = \frac{1}{\pi} \left[0 - \frac{\pi^2}{2} \right] + \frac{2}{\pi} \left[\frac{\pi^2}{2} - 0 \right]$$

$$a_0 = \frac{-\pi}{2} + \pi = \pi/2 \longrightarrow (b)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos x\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{0} (x \cos nx\,dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos nx)\,dx$$

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(P4)

$$a_n = \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left(\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right) = \frac{(-1)^n - 1}{\pi n^2}$$

Hence

$$a_n = \begin{cases} \frac{-2}{\pi n^2}\ ; \text{ if } n \text{ is odd number.} \\ 0\ ; \text{ if } n \text{ is even number} \end{cases} \rightarrow (c)$$

Now $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx.$

$$b_n = \frac{1}{\pi}\left\{x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right]$$

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

Hence the required fourier series is given as.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}$$

Ans

QUESTION NO 3: Solve the initial value problem.

$$y'' - 4y' + 13y = 8\sin 3x, \quad y(0) = 1 \text{ and } y'(0) = 2$$

SOLUTION:

$$y'' - 4y' + 13y = 8\sin 3x, \quad y(0) = 1 \text{ \& } y'(0) = 2 \rightarrow (1)$$

Associated Homogenous eq of (1) is

$$y'' - 4y' + 13y = 0 \rightarrow (2)$$

Change (2) into Auxiliary equation.

Put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

Use quadratic formula.

$a = 1, \quad b = -4, \quad c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{}$$

$$\frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$m_1 = 2+3i$

$m_2 = 2-3i$

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

let

$y_p = A \cos 3x + B \sin 3x \rightarrow$ ④

Diff w.r.t 'x'

$y_p' = -3A \sin 3x + 3B \cos 3x$

Again Diff w.r.t to 'x'

$y_p'' = -9A \cos 3x - 9B \sin 3x$.

Put in ①

$\Rightarrow (-9A \cos 3x - 9B \sin 3x) - 4(-3A \sin 3x) +$

$3B \cos 3x) + \cancel{B}(A \cos 3 + B \sin 3x) + B \sin 3x.$

$\Rightarrow (4A-12B)\cos 3x + (4B+12A)\sin 3x = 8\sin 3x.$

Comparing co-effient.

$\sin 3x \Rightarrow 4B+12A = 8 \rightarrow (a)$

$\cos 3x \Rightarrow 4A-12B = 0 \Rightarrow 4A = 12B$

$\rightarrow \boxed{A = 3B} \rightarrow (b)$

Put (b) in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = \frac{1}{5}} \rightarrow (c)$

~~put (c)~~ Eq put c Eq b

$a = 3/5 \rightarrow (d)$

put (c) & (d) in (A)

$y_P = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$

The G. Solution is

$y = y_c + y_P$

(P.1)

$$y = e^{2x}(c_1\cos 3x + c_2 \sin 3x) + 3/5 \cos 3x + \frac{1}{5}\sin 3x$$

$\longrightarrow$ (C)

Now we need to find the values of c_1 & c_2 for this

Put $x=0$ & $y=1$ in (C)

$$1 = e^{(2)(0)}(c_1\cos 3(0) + c_2 \sin 3(0)) + 3/5 \cos 3(0) + \frac{1}{5}\sin(3)(0)$$

$$1 = (c_1(1) + c_2(0)) + 3/5(1) + \frac{1}{5}(0)$$

$$1 = c_1 + 3/5$$

$$c_1 = 1 - 3/5$$

$$c_1 = 2/5 \rightarrow ☆☆$$

Now Diff 'c' with "x"

$$y' = c_1(2e^{2x}\cos 3x - 3e^{3x}\sin 3x) + c_2(2e^{3x}\sin 3x$$
$$+ 3e^{2x}\cos 3x) - 6/5 \sin 3x + 3/5 \cos 3x \rightarrow (D)$$

Put $y' = 2$, $x = 0$ in (D)

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x$$
$$+ 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + 3/5 \cos 3x.$$

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put $y' = 2$, $x = 0$.

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) + c_2(2e^{2(0)}\sin 3(0)$$

$$+ 3e^{2(0)}\cos 3(0)) - \frac{6}{5}\sin 3(0) + 3/_{15}\cos 3(0).$$

$$2 = c_1(2) + c_2(3) - 0 + 3/_{15}$$

$$2 = 2c_1 + 3c_2 + 3/_{15}$$

Put $c_1 = 2/15$

$$2 = 4/_{15} + 3c_2 + 3/_{15}$$

$$2 = 7/15 + 3c_2$$

$$3c_2 = 2 - 7/15$$

$$3c_2 = 3/15 \rightarrow ★★★.$$

Put ★★ & ★★★ in c.

$$y = e^{2x}(2/_{15}\cos 3x + 3/_{15}\sin 3x + 3/_{15}\cos 3x$$

$$+ \frac{1}{5}\sin 7x.$$

$$y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{15}\cos 3x + \frac{1}{5}\sin 3x \quad \text{Ans.}$$

QNo4: Solve

$$(D^2 - DD')z = \cos x \cos 2y$$

Solution:

It is already in symbolic form.

$$(D^2 - DD')z = \cos x \cos 2y \rightarrow (a)$$

Put A.E $D^2 - DD' = 0$

As we know

$$\frac{D}{D'} = m \quad \text{i.e } D = m, \quad D' = 1$$

$$\Rightarrow \quad m^2 - m = 0$$

$$m = 0, 1$$

There fore $\quad C.F = f_1(y) + f_2(y+x)$

From eq (a)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cos 2y$$

As

$$2\cos A \cos B = \cos(A+B) + \cos(A-B).$$

P13

$\therefore$ $C.F = f_1(y-x) + x f_2(y-x)$

$$PI = \frac{1}{D^2+2DD'+D'^2}\left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')^2}\left[2(y-x)+\sin(x-y)\right]$$

By General Method

$m_2 = -1$; $y-x=c$

$$= \frac{1}{D+D'}\left[\int [2c+\sin(-c)]\,dx\right]$$

$$= \frac{1}{D+D'}\left[2cx-(\sin c)x\right]$$

Rapleing by $y-x$

$$= \frac{1}{D+D'}\left[2x(y-x) - x\sin(y-x)\right]$$

Again put $y-x=c$

$$= \int (2xc - x\sin c)\,dx$$

$$\Rightarrow cx^2 - \frac{x^2}{2}\sin c$$

Replacing by $y-x$.

$$= x^2(y-x) - \frac{x^2}{2}\sin(y-x)$$

$$= x^2y - x^3 + \frac{x^2}{2}\sin(x-y).$$

Hence the required Solution is

$$z = C.F + P.I = f_1(y-x) + xf_2(y-x) + x^2y - x^3 + \frac{1}{2}x^2 \sin(x-y).$$

x ———— x ———— x.